

Where Do Dimensions Come From?

Are "dimensions" facts of nature waiting to be discovered, or are they artifacts of our imagination waiting to be invented?

Physicists are in no doubt about how they think of the world about them: "It seems inevitable that we should speak in terms of some definite theoretical model of the world of experience. There appears, however, to be no meaning in supposing there to exist a unique final model that we are trying to discover. We construct a model, we do not discover it" (McCrea 1983, p. 211). The idea of length is a theoretical model for an attribute of an object. Length does not exist on its own in nature, we invented it because it suits our purposes.

The idea of length is operationalized by means of devices such as rulers. Rulers are always imperfect representations of the idea of length. They are inaccurate and imprecise, but we use them because they are good enough for our purposes. Of course, every length-measuring process must be regulated to insure that the resulting number fits with our idea of length. Bending, breaking, or otherwise misapplying the ruler still produces numbers, but not numbers that fit our idea of length.

Length is apparent to us because it is visible, but what about temperature? We want to think about heat in the same way we think about length, as linear quantities. But we don't see heat in this way. Consequently we convert heat to length, or length-like numbers, by thermometers. Now we can think about and manipulate temperature in just the same way that we do length. Representation of abstract ideas requires visualization: rulers meet our need to think in a well-controlled, uniform way.

Educational tests must operate in the same way, if we want to make sense of them. We use our imaginations to invent a construct, math ability, that suits our purposes. This construct is our dimension. We express it in terms of an abstract item hierarchy: addition, subtraction, multiplication, division. We operationalize it in a math test. But is this dimension useful?

We discover whether our invented dimension, our construct, has any meaning and utility beyond our own imagination by looking for confirmation and contradiction of our intentions. We analyze the responses to our test. Do the item difficulties correspond with our intended hierarchy? Do individual items maintain their locations, i.e., do they fit? Are

noticeably different persons positioned at noticeably different locations on the dimension, i.e., separated, in a way that suits our purposes? Contradictions and deficiencies lead us to re-express our construct and revise our operationalization of it. Perhaps it would be more useful for our purposes for addition, subtraction, multiplication, and division each to have its own dimension, but that would lead to four measures. We must choose: Are four measures more useful or more confusing than one? Theory can't answer this, only practice can. If we have only one decision to make, then we want only one measure to base it on. If that one decision is actually a series of smaller decisions, then for each of those we want only one measure. Discovering that an individual's height is multidimensional with head dimension, torso dimension, and leg dimension is of no help, and so ignored for most purposes involving subject heights.

We know that perfection in conceptualization and operationalization will never be reached. A yardstick is not perfectly unidimensional, nor perfectly precise, nor perfectly accurate. But it is good enough for our purposes. So the ruler we construct from test responses falls short in just the same way. In both cases, we maintain the meaning of our dimension by careful use and maintenance of the ruler. We screen out and investigate errant measurements, misapplications, inconsistent results, and warped test instruments. We insist that only measures in useful accord with our invented dimension have the meaning we impute to the numbers. If no such measures are found, our dimension is useless, however conceptually sound it may be. If such measures are found, then they and only they suit our purposes, and the dimension is useful, however unrefined it may be.

Further thought and investigation will always reveal that our current idea of any dimension, "length," "temperature," "math ability" is deficient, and its operationalization by our ruler is defective. Progress requires that we be prepared to base our actions on what we can usefully achieve now, rather than on the perfection of the infinitely distant future.

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W.H. McCrea (1983) Introductory Remarks. *Phil. Trans. R. Soc. Lond. A.* 310:211-213.

