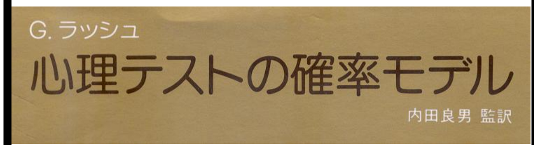
Georg Rasch in Japanese



RASCH MEASUREMENT

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Introduction to Georg Rasch's 1972 Retirement Lecture

Please remember, as you read our translation of Georg Rasch's *"Objectivity in Social Sciences. A Method Problem"* on page 1252 of this *RMT* that this is Rasch's manuscript for his Retirement Lecture and probably not a formal Journal paper. The language is at points obscure, and there are a few trivial errors (e.g., in the analysis of the data in Table 5) that were corrected when Rasch gave his lecture and would have been corrected if Rasch had taken the time to rewrite it as a paper. During the translation of the manuscript into English, we have been tempted to correct the errors and improve the language. We have (almost) not succumbed to the temptation, assuming that the reader prefers the pure Rasch version despite its shortcomings and will be able to read "between the lines" where that is required.

The paper itself is interesting for many reasons. Today, the majority of researchers using Rasch models remember him primarily for his contributions to objective measurement. The Retirement Lecture shows you Georg Rasch the statistician as Danish statisticians of the time remember him. The lecture mentions the Rasch model for dichotomous items in passing but is more concerned with situations requiring comparison of groups rather than persons and there is no discussion of measurements as such. The notion of *specific objectivity* was very important to Rasch. In the lecture, specific objectivity is a methodological rather than a measurement issue. As always, the frames of reference contain agents, objects and reactions, but the reactions depending on these are probabilities, not outcomes on stochastic variables. Given this set-up, Rasch talks about specific-objective estimation of parameters in probabilistic (and therefore statistical) models.

In Sections 2-10 Rasch revisits the deterministic case and Newton's Second Law. He was just setting the scene here, and there is probably nothing new to those who are well versed in the theory of Rasch models.

Section 11 uses the deterministic framework for a discussion of the dependence of production on capital and labour. This is the weakest part of the manuscript since it is a purely academic exercise with no data. My guess is that the only reason why this was included was that he was criticised (with considerable justification) for not having done anything for econometrics and economic statistics during his years as Professor at the Dept. of Statistics at the Institute of Economics.

Sections 12-15 are to me the most interesting sections. Here he discusses the logistic regression model and shows that specificly objective estimation of the parameters of such models is feasible. He never uses the term "logistic regression", and I remember thinking that he was "just" reinventing an already existing model when I heard him give this lecture. Rereading the manuscript after all these years, I can see that the point of the paper is that he discusses what we today would refer to as conditional logistic regression, pointing out that specific-objective estimation requires *conditional* logistic regression.

In the remaining sections, Rasch discusses the possibilities of extending the static frames of reference to dynamic frames and stochastic processes, and discusses data on development of wages over time and longitudinal data on mortality rates. Extending the frames of reference in this way was very important to Rasch and it was one of the issues that he intended to pursue in his years as Professor Emeritus. The Sections read more as an illustration of the problem and a statement of intent than anything else. Comparing these sections to the section on dynamic models in his 1977 paper on specific objectivity reveals that in this one respect he was perhaps less than successful, but it is still of interest to see what he was thinking about.

Svend Kreiner

June, 2010

Rasch, G. (1977). Specific Objectivity: An Attempt at Formalizing the Request for Generality and Validity of Scientific Statements. See www.rasch.org/memo18.htm

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Rasch Conference

Probabilistic models for measurement in education, psychology, social science and health

June 13-16, 2010, FUHU Conference Centre, Copenhagen, Denmark

Sunday, June 13th, 2010

17.00 – 19.00 Registration & opening reception

Monday, June 14th, 2010

9:00 – 9:20 Opening of conference – The meeting hall (second floor)
John Brodersen, Copenhagen University; Mette Madsen, Head of Institute of Public Health, Copenhagen University; Tina Nedergaard, Minister of Education, Denmark

Period: 1 9:30 – 10:30
A: The meeting hall (second floor)
Track: Education Historical perspectives on educational assessment. Chair: Tine Nielsen

"Application and Reception of Rasch Measurement in International Large-Scale Assessments of Educational Achievement. A Historical Perspective" Martin Goy, Wilfried Bos, Heike Wendt

"The Rasch model used in development of the National Tests in Denmark" Jakob Wandall

B: Auditorium 1 (first floor)
Track: Health Sample Size and Measurement Models. Chair: Stefan Cano

"Towards sample size determination for IRT analysis for the comparison of two groups of patients" Véronique Sebille, Sarah Amri, Jean-Benoit Hardouin

"Quantifying health-states: Discrete choice modeling and its relationship to fundamental measurement" Paul Krabbe

C: Auditorium 2 (basement)
Track: Psychometrics & Statistics Analysis of response behavior. Chair: Mounir Mesbah

"Using a theorem by Andersen and the dichotomous Rasch model to assess the presence of random guessing in multiple choice items" David Andrich, Ida Marais, Stephen Humphry

"Assessing and quantifying inter-rater variation for dichotomous ratings using a Rasch model" Jørgen Holm Petersen

Period: 2 11:00 – 12:30
A: The meeting hall (second floor)
Track: Psychometrics & Statistics Inference: DIF 1. Chair: Svend Kreiner

"Investigating the Sensitivity of Anchor Item Method for Detecting Differential Item Functioning relative to Sources of DIF in a Translated Test" Goran Lazendic, Saw Choo Teo

"Item Analysis via Three-Level Hierarchical and Cross-Classified Models" Yuk Fai Cheong

"Three-step latent variable regression with differential item functioning" Janne Petersen, Karen Bandeen-Roche, Klaus Groes Larsen, Ove Andersen, Esben Budtz-Jørgensen

B: Auditorium 1 (first floor)
Track: Health Developing Scales that fit Rasch Models. Chair: John Brodersen

"Building face and content validity into health outcome measures in the context of Rasch analysis" Steven McKenna, L. C. Doward, J. Twiss

"Application of Rasch analysis in the development of the PRIMUS QoL Scale." Lynda Caroline Doward, Steven McKenna, J. Twiss, Benjamin Eckert

"Rasch analysis of the Dermatology Life Quality Index (DLQI)" James Twiss, Steven McKenna, Lynda Caroline Doward

C: Auditorium 2 (basement)
Track: Education Issues in Scale Development. Chair: Jack Stenner

"Evaluation of Equating in the International School Assessment (ISA) program" Yan Bibby

"A Rasch Measure of Form 'Constancy of Letters and Numbers, Letters in Words and Numbers in Calculations for Young Children' " Russell F. Waugh, Janet Richmond

" 'What do Grade 6 Pupils and Teachers Know about HIV and AIDS in 14 Sub-Saharan Africa Countries?' – Design of the SACMEQ HIV-AIDS Knowledge Test and Results in the Rasch Measurement Framework." Stéphanie Dolata

14:00 – 15:00 Keynote – The meeting hall (second floor)

"Rasch's contribution to an understanding of physical measurement" David Andrich

Period: 3 15:30 – 17:00
A: The meeting hall (second floor)
Track: Psychometrics & Statistics Measurement issues 1. Chair: Thomas Salzberger

"The Quest for Invariant Measurement" George Engelhard

"In Pursuit of Individual Score Validity: The Wariness Index" Carl Hauser, G. Gage Kingsbury

"Differential item Functioning (DIF) on Gender, Time, and Country in the SACMEQ III Research Study" Mioko Saito

B: Auditorium 1 (first floor)
Track: Psychology Psychometric properties of psychological measurement scales. Chair: Henriette Kirkeby

"Using Rasch measurement to validate the Big Five Factor Marker personality questionnaire for a Japanese population" Matthew Thomas Apple, Peter Neff

"The Effect of Local Item Independence on Assessments of Multidimensionality: An Example From the Mayer-Salovey-Caruso Emotional Intelligence Test (MSCEIT)" Andrew Eliot Maul

"Measuring Psychosocial Learning Climate – an analysis of the psychometric properties of a scale using Swedish adolescent data" Daniel Bergh

C: Auditorium 2 (basement)
Track: Psychometrics – Health Computer-adaptive Testing. Chair: Steven McKenna

"Analysis of Information and Targeting in Graphical Loglinear Rasch Models" Svend Kreiner, John Brodersen

"Unproctored Internet Test Verification: Using Adaptive Confirmation Testing" Guido Makransky

"Development of a Rasch-based item bank to measure fundamental aspects of work disability in patients with musculoskeletal diseases" Evelyn Mueller, C. Frey, E. Prinz, J. Bengel, Markus Wirtz

Tuesday, June 15, 2010

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A: The meeting hall (second floor)
Track: Psychometrics & Statistics Measurement issues 2. Chair: Jørgen Holm

"An investigation of the unit of measurement of an agree-disagree versus a disagree-agree response scale – a cautionary note" Thomas Salzberger

"Combining time and correctness in the scoring of performance on items" Gunnar R. Bergersen

"Causal Rasch Models" Jack Stenner, Mark H. Stone, Donald S. Burdick

B: Auditorium 1 (first floor)
Track: Sociology & Marketing Psychometric properties of measurement scales. Chair: Guido Makransky

"Psychometric properties of the daily spiritual experience scale (DSES) analyzed through the Rasch method" Acacia Lima Oliveira, M. Kimura, Neusa Sica da Rocha

"Evaluating the Psychometric Quality of a Racial Attitudes Index with a Rasch Measurement Model" Khaya D. Clark, George Engelhard

"Measuring the specialness of confectionery: a Rasch model approach in affective engineering" Fabio Camargo, Brian Henson

C: Auditorium 2 (basement)
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"Beyond traditional psychometric methods: Can Rasch help the DASH?" Stefano Cano, Louise Barrett, John Zajicek, Jeremy Hobart

"A review of methods to test multidimensionality in the FIM scale" Valeria Caviezel

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"Rasch goes Open Source: The R package "eRm" for the computation of extended Rasch models" Patrick Mair, Marco Maier

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"Locating objects on a latent trait using Rasch analysis of experts' judgments" Tom Bramley

"Developing and validating the scoring of innovative items using Rasch fit statistics" Kirk Becker, John De Jong

C: Auditorium 2 (basement)
Track: Health Patient Symptoms and Scale Measurement. Chair: John Brodersen

"What Lies Beneath: Interpretation Issues in the Analysis of a Popular Fatigue Scale" Anita Lynne Slade, John P. Zajicek, Wendy M. Ingram, Stefan J. Cano, Roderick Freeman, Jeremy C. Hobart

"A Rasch analysis of the Comprehensive ICF Core set for low back pain" Cecilie Røe

"The construction of an *ad hoc* scale to study the natural development of symptom burden in patients with type 2 diabetes and its relation to glycaemic control" Volkert Siersma, Paolo Eusebi

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"Psychosystems: An alternative approach to psychometric theory" Denny Borsboom

Period: 6 15:30 – 16:30
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Track: Health Invariance in Health Measurement. Chair: Curt Hagquist

"Spatio-temporal Rasch analysis of Quality of life outcomes in the French general population" Jean-Benoit Hardouin, Alain Leplège, Etienne Audureau, Joël Coste

"Coordinating cross-cultural comparisons: Lessons from Friedreich's Ataxia" Roderick Freeman, Stefan Cano, Anita Slade , Martin Delatycki, Geneieve Tai, Jeremy Hobart

B: Auditorium 1 (first floor)
Track: Education New Perspectives on Educational Assessment. Chair: Alain LePlege

"Objective Classroom Assessment Using the Rasch Model" Brian Bontempo

"Commensurable system-wide and teacher assessment" Gordon Cooper, Stephen Humphry

C: Auditorium 2 (basement)
Track: Psychometrics & Statistics Measurement Issues 3. Chair: Volkert Siersma

"Investigation of Rasch Measurement Precision depending on Number of Dichotomous Items" Anatoly Andreyevich Maslak

"Development of Rasch based adaptive assessment procedures to measure 'functionality in every day life' in rehabilitation patients with musculoskeletal diseases" Markus Antonius Wirtz (presented by Maren Böcker)

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A: The meeting hall (second floor)
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"A systematic review of Basic-Instrumental Activities of Daily Living revised through Item Response Theory methodology: Entering a new generation of instruments assessing functional status?" Robert A. Fieo, Ian J. Deary, John M. Starr

"Visual Analogue Scales: sensitivity or deception?" Paula Kersten, Alan Tennant, Peter John White

"Implications on person measurement of collapsing response categories – An illustration using the SDQ-impact-scale" Curt Hagquist

B: Auditorium 1 (first floor)
Track: Health Dimensionality of Well Established Questionnaires. Chair: Anita Slade

"Measurement properties of the WHOQOL-BREF in alcoholics using the Rasch model" Neusa Rocha, Felix Kessler, Sibele Faller, Daniela Benzano, Flavio Pechansky

"Fitting a Mixed Rasch Model to Nottingham Health Profile in Turkish Population" Selcen Yuksel, Atilla Halil Elhan, Derya Oztuna, Ayse A. Kucukdeveci, Sehim Kutlay

"Evolution not revolution: Scaling new heights in cognitive performance measurement" Jeremy Hobart, Stefan Cano

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"Checking or testing unidimensionality under Rasch or Classical methods" Mounir Mesbah

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"Comparison of three methods for the analysis of longitudinal Patient Reported Outcomes" Myriam Blanchin, Jean-Benoit Hardouin, Tanguy Le Neel, Gildas Kubis, Véronique Sébille

"Attention Deficits Questionnaire (ADQ): a self-rating scale for the neurorehabilitation" Effi Volz-Sidiropoulou, Siegfried Gauggel

B: Auditorium 1 (first floor)
Track: Psychometrics and Statistics Statistical Theory. Chair: David Andrich

"The Rasch model as a statistical model: Erling B. Andersen's contributions to the theory of Rasch models" Karl Bang Christensen

"The genealogy of the Rasch models from origin to present" Henrik Svend Albeck

"Testing the Rasch model: Optimal sample size determination for the Andersen-Test" Rainer W. Alexandrowicz

C: Auditorium 2 (basement)
Track: Education – Rasch Modeling's Potential for Informed Practice. Chair: Gordon Cooper

"Connecting measurement with substantive theory: an attempt to locate threshold concepts in the multiplicative conceptual field" Caroline Long, Tim Dunne

"Using Rasch Model to Communicate about Factors which Encourage and Discourage University Lectures to Infuse Education for Sustainable Development (EfSD) into Teaching" Mohd Ali Samsudin, Sharifah Norhaidah Syed Idros

"Rasch, Maxwell's Method of Analogy, and the Chicago Tradition" William P. Fisher

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"Psychological Measurement Between Physics and Statistics" Klaas Sijtsma

Period: 9 15:30-16:30

A: The meeting hall (second floor)
Track: Psychometrics & Statistics Inference: DIF 2. Chair: Ida Marais

"A New Method for Detecting Differential Item Functioning in the Rasch Model" Carolin Strobl, Achim Zeileis, Julia Kopf

"The evaluation of the impact of uniform and nonuniform differential item functioning on Rasch measure" Silvia Golia

B: Auditorium 1 (first floor)
Track: Education Invariance in Language and Fluency Assessment. Chair: Caroline Long

"Validity of Elementary school fluency measures for reading and writing modeled as Poisson counts" Peter Denton MacMillan

"An investigation of Differential Item Functioning in the Michigan English Language Assessment Battery Listening Test" S. Vahid Aryadoust, Christine Goh

C: Auditorium 2 (basement)
Track: Health Item Banking. Chair: Linda Doward

"Developing an item bank for Emotional Vitality: a methodological overview" Skye Pamela Barbic, Nancy E. Mayo, Lois Finch

"On the way to the NeuroCAT: Development and initial evaluation of the Aachen-ADL-Item Bank" Maren Böcker, Markus Wirtz, Nicole Eberle, Siegfried Gauggel

16:30 – 17:30 Closing Keynote – The meeting hall (second floor)

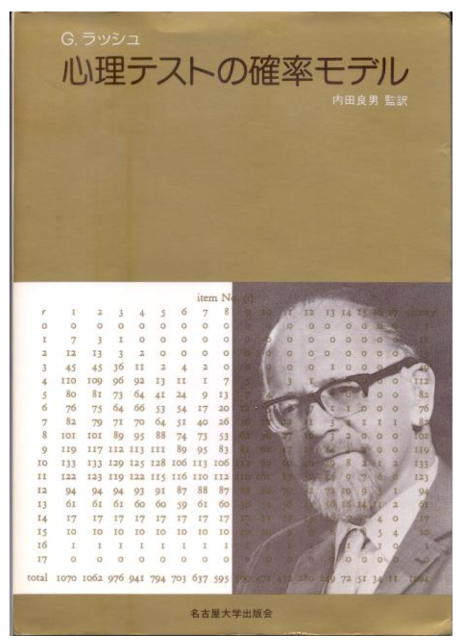
"Rasch Models – Past, Present, and Future" Svend Kreiner

The Japanese Translation of Rasch's "Probabilistic Models"

A Japanese translation of Georg Rasch's "Probabilistic Models for Some Intelligence and Attainment Tests" was published in August 31, 1985 by the University of Nagoya Press with permission from the University of Chicago Press, the copyright-owners at that time, through Tuttle-Mori Agency, Inc., Tokyo. The book has 237 pages and ISBN 978-4-930689-35-1.

The Japanese title is "Shinri Tesuto no Kakuritsu Moderu" (Probability Models for Psychological Tests).

The translation was supervised by Yoshio Uchida, Professor at Aichi Gakuin University and Professor Emeritus of the University of Nagoya, Education Department. He specialized in statistics.



The translation was performed by:

Foreword (by Benjamin D. Wright) and Preface: Prof. Motomichi Gotoh, later Professor Emeritus, University of Nagoya.

Chapter 1, 2: Prof. Yoshitaka Makino, later Professor at Chukyo University School of Psychology

Chapter 3, 4: Prof. Toru Masui, later believed to be Chief, Division of Bioresources Research, National Institute of Biomedical Innovation

Chapter 5, 6: Prof. Toshio Uchida, later a Professor at Chubu University

Chapter 7, 8: Prof. Naohito Chino, later Professor, Department of Psychology, Aichi Gakuin University

Chapter 9, 10: Prof. Takashi Murakami, later Professor in the Department of Educational Psychology, Nagoya University

Appendix and Afterword (By Benjamin. D Wright): Prof. Hideo Tsujimoto, later at the Department of Psychology, Osaka City University

The book is now out-of-print, but there are copies in 118 Academic Libraries in Japan, including Sophia University and the Hokkaido University of Education. Copies are currently unavailable, but a tentative price is 3,675 Yen (= $42), and several copies are privately-owned by our Japanese colleagues.

Thomas Salzberger, Lina Wøhlk-Olsen, and many colleagues in Japan provided this information. *Thank you!*
Picture of book's cover is courtesy of Thomas Salzberger.

Pacific Rim Objective Measurement Symposium 2010

Kuala Lumpur, Malaysia

29 June – 1 July 2010

Plenary Sessions:

Dr. Jack Stenner: Causal Models

Dr. Margaret Wu: School Based Assessment

Dr. Anthony Zara: Certification & Licensure

Professor George Engelhard: Performance Assessment

Dr. Noor Lide Abu Kassim: Standards & Standard Setting

Parallel Sessions:

A.Y.M. Atiquil Islam, Mahbubul Haque, Tunku Badariah Tunku Ahmad & Mohamad Sahari Nordin: An Empirical Study on Students' Intention to Adopt Online Research Database

A.Y.M. Atiquil Islam, Noor Lide Abu Kassim & Mahbubul Haque: Development and Validation of Technology Acceptance Scale for IIUM Students using Rasch Analysis

Adidah Lajis & Normaziah Abdul Aziz: Assessment of Learners' Understanding: The Design and Evaluation of Node Link Scoring Technique

Ahmad Zamri& Noor Lide Abu Kassim: Setting Performance Standards in Mathematics for Form 2 Students: Application of the Bookmark Method

Akbariah Mohd. Mahdzir, Nadiah Ahmad Supian& Ling Swee Eng: The Application of Rasch Model in the Development of Voting Pattern Indicator (VPI) Instrument Amongst the Young

Aswati Hamzah, Ahmad Zamri Khairani & Nordin Ab Razak: Defining Spiritual Disposition Scale of Undergraduates Students

Azlan Mohamed Zain: Faculty's Adoption of Learning Management System for E-learning: An Extended Technology Acceptance Model

Bakare Kazeem Kayode & Che Noraini Hashim: Managing Occupational Stress Among Private Islamic School Administrators and Senior Teachers in the Klang Valley

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Brian Doig: The TIMSS Viking Rubrics: Are they worth the effort?

Chang Mingchiu & Tzou Hueying: The Performance of Imputation on the Detection of the Multidimensionality

Chien Tsair-Wei & Wang Wen-Chung: Reducing Burdens with Computerized Adaptive Assessment on an Animation Satisfaction Questionnaire

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David Beglar: A Longitudinal Study of Changes in Willingness to Communicate in a Foreign Language

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Wang Wen-Chung, Qiu Xuelan, Jin Kuan-Yu & Lei Wang: Analyses of Examinee-Selected Items

William Koh Siaw Yen & Ong Saw Lan: Assessment of Psychometric Properties of Items using Structural Equation Modeling and Rasch Measurement Model within the Framework of an Instrument (SPAQ1-Students Parent's Actions Questionnaire) of Students'

Yahya Ibrahim Saleh & Mohammad Khan Jamal Khan: Comparative Analysis between Nigeria and Malaysian National Health Insurance Scheme Reform and Health Care Delivery System: Perspective on Safety and Health Management

Yang Ya-Huei, Lin Chuan-Ju & Hung Pi-Hsia: The Factors influencing the difficulty of English Listening Tests

Zainariah Mohd Nor, Nik Maryam Idris, Mohd Zali Mohd Nor & Azrilah Abd Aziz: Measuring Students Performance for English for Science and Technology – A New Alternative for Malaysian Secondary School

Rasch-related Coming Events

July 26 - Nov. 20, 2010, Mon.-Sat. Online course: Introduction to Rasch Measurement of Modern Test Theory (Andrich, RUMM2030) www.education.uwa.edu.au/ppl/courses/introduction

Aug. 7, 2010, Sat. How to publish papers in international journals using Rasch Analysis and a Workshop Course on Winsteps, Taiwan www.healthup.org.tw/rasch/English.htm

Aug. 20 - Sept. 17, 2010, Fri.-Fri. Rasch - Core Topics (M. Linacre, Winsteps), www.statistics.com

Aug. 23, 2010, Mon..Online Rasch courses at the University of Illinois Chicago begin, (E. Smith), www.rasch.org/onlineuic.htm

Sept. 1-3, 2010, Wed.-Fri. ICOM 2010 International Conference on Outcomes Measurement, Bethesda MD, www.esi-bethesda.com/icom2010

Sept. 1-3, 2010, Wed.-Fri. 13th IMEKO International Measurement Confederation, London, UK, imeko.iopconfs.org

Sept. 15-17, 2010, Wed.-Fri . Introduction to Rasch

Sept. 20-22, 2010, Mon.-Wed. Intermediate Rasch

Sept. 23-24, 2010, Thur.-Fri. Advanced Rasch (A. Tennant, RUMM), Leeds, UK, www.leeds.ac.uk/medicine/rehabmed/psychometric

Oct. 27-30, 2010, Wed.-Sat. ISOQOL 17 International Society for Quality of Life Research, London, England www.isoqol.org

Nov. 26, 2010, Fri. V Workshop "Modelos de Rasch" Canary Islands (Spanish), www.institutos.ull.es/view/institutos/iude/Inicio/es

Apr. 8-12, 2011, Fri.-Tues. AERA Annual Meeting, New Orleans, LA, www.aera.net

Objectivity In Social Sciences: A Method Problem

By **Georg Rasch,** Professor at the University of Copenhagen.

Retirement lecture, 9 March 1972. Translated by Cecilie Kreiner [*]

1. *The necessity of relationships being generally applicable*. Whether the completion of a specific task, in which there is correlation between financial, demographical, sociological and/or other variables, is successful naturally depends on whether the ratios at your disposal actually fit and are adequate under the requirements of the task.

The relationships can also be extracted through purely theoretical reasonings or be entirely or partially defined based on empirical data. But in order to be able to use them freely, you have to make sure that they are sufficiently generally applicable.

There is assuredly reason to admire, for example, the launching of rockets to the moon, but the success of such projects is based on the fact that they are extremely carefully thought out and planned in all details, using a number of physical laws in the fields of statics, dynamics, electronics etc.: the technicians rely on *the basic laws applying anywhere and anytime within their field of work*. Otherwise, they would not be able to build or construct anything without risking the worst.

And should the worst still come to happen – and occasionally it does – then we say that "a technical error has been made", which means that the physical laws that the construction should be based on have not been fully respected, whether due to carelessness or a lack of knowledge, possibly far back in a corner of physics that was not sufficiently explored.

If directly available observations are to be useful, whether for inclusion in the continued development of the theory or directly in practice, it is not enough that the relationships extracted from them offer an *ad hoc* description, however adequate, of the existing data – that it only guarantees to be applicable "here and now" – ***the derived relationships must be general.***

2. *Explanation of a physical law: incomplete induction and circular arguments?* But is such a thing even possible?! You can never have more than a finite number of observations at your disposal, and here it is expected that it should be possible to extract something general from them!

No, of course *that* is not possible – it would be a purely formal ideal demand. But let us see how far you can get by subjecting one of the simplest physical laws to a careful analysis in order to uncover how it could, in principle, be explained.

The set-up of the equations for how solid bodies move depend, for example, on the following law (Newton's second law): *the force F that gives a solid body an acceleration of its velocity of A is proportional to the product of the acceleration and the mass of the body M:*

$$F = G \cdot M \cdot A \tag{1}$$

where the value of the proportionality constant G depends on the units in which F, M and A are expressed.

Controlling the accuracy of such a ratio is seemingly fairly simple, at least within certain limits: take a collection (m) of solid bodies with entirely different masses that move in the same direction relative to Earth and expose each of them to a number (n) of mechanical instruments that urge them in their direction of momentum, however with widely different forces. In each of the $m \cdot n$ experiments, the acceleration is measured and you check whether the ratio fits – at least for *those* bodies and *those* instruments.

Yes, if only it were that simple. Yet it is not, and that is primarily due to the fact that, in the experiments described above, you had to know in advance the force which each of the instruments exerts, as well as the mass that each of the bodies contains.

And now we are touching on one of the controversial subjects in classical physics: what is mass? And what is force? Or if it is not possible to find out *what* the one and the other *are*, how can you *measure* them?

It is question on which volumes are still written[1]. And they seem to conclude that if you knew what mass was, it would be possible to say what force is – and *vice versa!*

3. *Data structure and simultaneous inclusion of mass and force*. It could sound as if you were stuck, but it really just means that the experiment had to be based on the fact that neither the mass of the bodies nor the force of the instruments was known from the outset – or put even more agnostically: that you do not even know whether there is anything at all that can be termed mass and force, and that therefore you do not have a relationship of either that the experiment claimed to check. That the only thing you actually *know* is what you *observe*, i.e. that when a solid body L_i moves at a certain (temporary) velocity V_i relative

[1] For example Jammer, Max (1957) and (1961).

to, for example, the Earth and receives a thrust in the direction of the movement by a, for that purpose, suitable instrument I_j, then the velocity of the body is changed with an acceleration A_{ij} *that is measurable* (that this is possible is therefore a prerequisite).

The experiment outlined above then results in $m{\cdot}n$ observations A_{ij} that can be collected in rectangular array:

(2)

		Instruments				
		I_1		I_j		I_n
	L_1	A_{11}	...	A_{1j}	...	A_{1n}
	.					
	.	....	...	....	...	....
Solid	.					
	L_i	A_{i1}	...	A_{ij}	...	A_{in}
bodies	.					
	.	....	...	....	...	....
	.					
	L_m	....		A_{mj}	...	A_{mn}

Concerning these results, physicists assure me that if you really performed such an experiment – which, by the way, would never occur to them since, through other experiences, the outcome would be known in advance – then the accelerations would form a nice multiplicative system: each row would be proportional to any other row, and each column to any other column, so that the acceleration A_{ij}, except a proportionality constant G, could be decomposed into a product of a row factor P_i and a column factor Q_j:

$$A_{ij} = G \cdot P_i \cdot Q_j \tag{3}$$

It can therefore, except for the constant, be described as the product of a parameter P_i for the body L_i, and a parameter Q_j for the instrument I_j. And thereby we will, via an empirical route, have found a relationship of the form (1), in which you should then have

$$Q_j = F_j \text{ and } \qquad P_i^{-1} = M_i \tag{4}$$

But precisely this way of writing the parameters – that is as Q_j itself, but the reciprocal value of P_i – requires an explanation in addition to (3) and its formal paraphrase into (1). It is given in two supplementary experiments – once again "thought experiments". From the first it follows that if you let two instruments I_j and I_k act immediately one after the other on the same body L_i, the effect is the same as if it were just one instrument with the parameter

$$Q_{(jk)} = Q_j + Q_k \tag{5}$$

or according to (4)

$$F_{(jk)} = F_j + F_k \tag{5a}$$

The second experiment shows that if two bodies L_h and L_i are attached together, they function as one body when influenced by an instrument I_j, but that the parameter $P_{(hi)}$ for the compound body satisfies the relationship

$$P_{(hi)}^{-1} = P_h^{-1} + P_i^{-1} \tag{6}$$

that is with the notation of (4):

$$M_{(hi)} = M_h + M_i \tag{6a}$$

Hereby it is first and foremost realised that the parameters that form parts of the law (1) as "mass" and "force" need not be individually defined, nor by each other; they can be included simultaneously through what can be determined, i.e. *the structure of the accelerations in the thought experiment* (2), *in which composite objects and instruments that act serially are included*.

To this may be added that in the additive formulas (5) and (6) for the parameters that form a part of (3), there is a motivation for attaching the terms "mass" and "force" on P_i^{-1} and Q_j respectively. The intuitive perception of the phenomenon "force" is not only that the effect increases with increasing "exertion of force", but also that it happens "quantitatively", i.e., that for example, the application of the same force twice in a row on the same body has the same effect as the "double exertion of force". Likewise, the intuitive perception of the phenomenon "mass" is, for example, that two identical bodies that are tied together have the "same inertia" – are equally difficult to move – as one body with "double the mass".

4. *Testing of hypothesis vs. incomplete induction*. Even though we have hereby placed the *terms* in the (*F, M, A*) constellation, we have as yet not explained (1) as a general law. Experiments with 20 or thousands of bodies, whether exposed to 7 or 7000 instruments, can still only produce *a finite number of observations*; and even though physicists claim all manner of experiences as explanation for the outcome of the thought experiment, their scope, however vast, is still limited. Therefore, even a very large experiment and/or a well-founded thought experiment does not explain in principle the ratio as *generally* valid. As implied in section 2: **general validity can simply not be achieved empirically.** Regardless of the scope, the documentation remains *an incomplete induction*.

Nevertheless, (1) forms part of the standard basis for the whole of classical physics and its technical applications. Then how can it be explained?

Laws such as (1), together with (5a) and (6a), can be viewed as deductively derived from already accepted theory, however that actually just moves the problem a step further back, unless you want to use the law in question as a touchstone for their premises. In any case, the result is that an empirical material of large or small scope can lead to the *assumption* that there is some kind of law regularity. In this case, that any body L_i can be given a parameter M_i, and that any mechanical instrument I_j can be given a parameter F_j, which together satisfy both the multiplicative acceleration relationship (1) and the additive relations (5a) and (6a).

This assumption can be *tentatively* elevated to *a general hypothesis*, which is then tested at every opportunity with many kinds of solid bodies and many kinds of mechanical instruments – partly directly, partly indirectly through their consequences, for example by actually launching and controlling rockets according to plan.

So: ***you do not prove a law*** such as (1) or its parameterised form (3), with or without the additive laws (5a) and (6a); ***observations inspire you to set it up as a hypothesis, which is then tested on a very wide basis.***

We have thereby answered the question posed in section 2 about the principled explanation of a law such as (1).

5. *Delimiting the field of validity.* While many kinds of tests have certainly strengthened your faith in the proposed hypothesis, they have also served to delimit its field of validity: it applies *within a certain frame of reference*, in which the bodies are solid, the instruments function solely mechanically, and in which the reactions are the accelerations of the bodies.

If the frame of reference is extended, the hypothesis may no longer apply. If, for instance, you kick 1 kg butter at 20 degrees centigrade, it will stick to your shoe, and if an instrument functions not only mechanically but also magnetically, objects made from stone and iron will react in quite different ways. And if other things beside accelerations are taken for reactions – for example velocities or positions, not to mention the colour and light reflection of the bodies – then (1) will, of course, cease to apply.

The principal thing is that, after a good start with apposite bodies, instruments and reactions chosen based on everyday criteria, you can attempt to extend the frame of reference in different directions, delimit the class of bodies and instruments to which the tested hypothesis applies, and in the end discover which physical qualities they must have in contrast to those to which the law does not apply.

Thereby you can gradually reach a clarification of the field of validity of the law.

6. *Comparisons within the frame of reference.* Next, let us take a closer look at the contents of the law (3).

First, we supposed that there were *m* solid bodies $L_1, \ldots, L_m$, but then we realised that generality demanded the inclusion of many more in the experiment. In fact, it would not be possible to stop after any given number: the set of bodies potentially on trial is *infinite* or, to put it plainly, the general law can only be formulated for an infinite set of bodies. We designate such a set L.

The same applies to the instruments: the law can only be formulated for an infinite set I.

Finally, with regard to the accelerations, the possible values form a set A, which must also be considered infinite since any positive real number is possible.

The frame of reference for the law in question is then the set of the three sets

$$[\mathrm{L,I,A}] \tag{7}$$

and the law itself is (3), in which *i* and *j* are indices, which are not presupposed numerable even though they are formally presented as numerals in the following; but hereby we are implying a specific, and thereby finite, set of data.

Considering (3) as valid for $i = 1, \ldots, m$, $j = 1, \ldots, n$ the corresponding parameters P_i and Q_j can be determined from the *A*s, after obtaining the proportionality factor *G*, which can be established by choosing the units for *P* and *Q* so that, for example,

$$P_1 = Q_1 = 1 \tag{8}$$

whereby we get

$$G = A_{11} \tag{8a}$$

The point in this banal observation is that P_i and Q_j cannot be determined absolutely but only relative to something else, in this case P_1 and Q_1 respectively.

L_i can thus only be estimated through comparison with another body in L, and I_i only through comparison with another instrument in I.

If the same instrument I_i is used for setting up the comparison between two random bodies L_h and L_i in L, then it is based on the two accelerations A_{hj} and A_{ij} thus expressed according to (3):

$$\frac{A_{hj}}{A_{ij}} = \frac{G \cdot P_h \cdot Q_j}{G \cdot P_i \cdot Q_j} = \frac{P_h}{P_i} \tag{9}$$

The result of this comparison has two obvious qualities:

a. *It is independent of all other bodies in* L, particularly of the other bodies in a relevant collection $L_1, \ldots, L_m$.

b. *It is independent of which instrument in* I *is used for setting up the comparison*, particularly of the other instruments in a relevant collection $I_1, \ldots, I_n$.

Similarly, two random instruments I_j and I_k in I are compared by means of the two accelerations A_{ij} and A_{ik} that they effect in the same body L_i, since

$$\frac{A_{ij}}{A_{ik}} = \frac{G \cdot P_i \cdot Q_j}{G \cdot P_i \cdot Q_k} = \frac{Q_j}{Q_k} \tag{10}$$

which is only dependent on the two instruments, but neither on the other instruments in I (cf. a.), nor on the body used (cf. b.).

7. *The specific objectivity of the comparisons*. All the possible situations for observations have now been defined by means of the frame of reference [L,I,A]: the bodies in L must be compared with regard to the accelerations (A) the instruments in I inflict on them. And the instruments are compared analogously.

That presupposes implicitly that the observations take place within an isolated system so that they are not affected by what goes on in the world outside. That is, neither by the position of the stars, lorries driving by or high political problems. However, it is also required that the design of the study – the necessary manipulation of bodies and instruments, as well as the registration of the accelerations – does not interfere in the observation situation.

This strictly isolated system is thus completely characterised by the frame of reference [L,I,A] and the corresponding parameters. Within this frame, all possible *A*s are potentially given data – in a relevant observation situation, A_{ij}; $i = 1, \ldots, m$; $j = 1, \ldots, n$ is the actual given data – while *the parameters P and Q are unknown, but they are the only unknown part of* [L,I,A]. The statements *a* and *b* then claim that, given the relationship (3) as **a fundamental foundation of the frame of reference, *the parameters for two random bodies can be compared based on what is known, i.e. observed accelerations, and the result is unaffected by everything unknown outside the frame of reference*.**

That the analogous situation applies to the comparison of instrument parameters is self-evident.

In this precise sense, we can term the comparisons "objective". However, in both science and daily debate, this expression is used to mean a number of things, and therefore I will tighten up the terminology by *terming the comparisons* ***specifically objective*,** that is specified by the frame of reference.

8. *Scalar latently additive differences.* The analysis of Newton's second law carried out here has its parallels in the fundamental laws of elementary classical physics, many of which are multiplicative like (1), and in several cases, they are followed up by analogies to the additive laws (5a) and (6a). However, regardless of whether the latter are found or not, the specifically objective comparison can be established through (1).

However, not only does this law generate such comparisons. It is possible that there are objects *O*, other than solid bodies, that came into contact with agents *A*, other than just moving mechanisms, thereby resulting in reactions *R*, other than just accelerations. Furthermore that, with regard to this relationship, *O*, *A* and *R* are characterised entirely by unidimensional – so-called scalar – real parameters *o*, *a* and *r*. Since *R* is considered determined by *O* and *A*, *r* has to be an unambiguous function of *o* and *a*:

$$r = r(o, a) \tag{11}$$

In the special case of the previous example,

$$r = o \cdot a \tag{12}$$

which logarithmically transformed can be expressed additively

$$\log r = \log o + \log a \tag{12a}$$

which yields, when used on m objects, n agents and $m \cdot n$ reactions,

$$\log r_{ij} = \log o_i + \log a_j\ , \quad i = 1,\ldots,m;\ j = 1,\ldots,n \tag{13}$$

or

$$\overline{r}_{ij} = \overline{o}_i + \overline{a}_j \tag{13a}$$

where the lines indicate the logarithmic transformation.

In this additive system, which is, of course, equivalent to the multiplicative system (12), o_h and o_i can be compared by

$$\overline{o}_h - \overline{o}_i = \overline{r}_{hj} - \overline{r}_{ij} \tag{14}$$

which applies to every j and is therefore a specifically objective statement. The analogous situation applies to comparison of two *a*s.

A handy control of the additivity, which at the same time determines the addends except for an additive constant, can be attained by forming the average over i and j respectively in (13a):

$$\overline{r}_{i\bullet} = \overline{o}_i + \overline{a}_\bullet, \overline{r}_{\bullet j} = \overline{o}_\bullet + \overline{a}_j \tag{15}$$

which yields when i is inserted (13a)

$$\overline{r}_{ij} = \overline{r}_{i\bullet} + (\overline{a}_j - \overline{a}_\bullet), \overline{r}_{ij} = \overline{r}_{\bullet j} + (\overline{o}_{ji} - \overline{o}_\bullet) \tag{16}$$

For fixed j, the difference $\overline{r}_{ij} - \overline{r}_{i.}$ will be constant so that $\overline{r}_{ij}$ plotted against $\overline{r}_{i.}$ gives points on a straight line with the slope 1. Analogously for $\overline{r}_{ij}$ plotted against $\overline{r}_{.j}$.

However, the same reasoning also applies if just r is dependent on o and a so that there are 3 functions

$$\overline{r} = f(r)\ , \quad \overline{o} = g(o)\ , \quad \overline{a} = h(a) \tag{17}$$

of r, o and a that satisfy the additive relationship

$$\overline{r} = \overline{o} + \overline{a} \tag{18}$$

In that case, we refer to the system [o, a, r] as a *latent additive* system, here presupposed *scalar*, and we now know that *such a system guarantees the possibility of specifically objective comparisons between objects and between agents*.

9. *Condition for latent scalar additivity*. Examining whether a system of scalar variables is latent additive is, in principle, rather simple by differentiating the equation equivalent to (18)

$$f(r) = g(o) + h(a) \tag{18a}$$

with regard to o and a respectively to obtain the two relationships

$$\frac{\delta r}{\delta o} \cdot f'(r) = g'(o)\ , \quad \frac{\delta r}{\delta a} \cdot f'(r) = h'(a) \tag{19}$$

Where $f'(r)$ is eliminated by division to obtain

$$\frac{\delta r}{\delta o} \Big/ \frac{\delta r}{\delta a} = \frac{g'(o)}{h'(a)} \tag{20}$$

which predicts that the relationship between the two partial differential quotients of the reaction function *r* must form a multiplicative system.

Whether it does form a multiplicative system can be examined by taking the logarithms and, by means of the technique outlined in (15) and (16), examining whether they form an additive system. If they do, you will at the same time determine *g'(o)* and *h'(a)* – except for a multiplicative constant – and can by means of integration form *g(o)* and *h(a)* – except for additive constants.

Since (20) is not only a necessary but also a sufficient condition for latent scalar additivity, the sum *g(o)* + *h(a)* must necessarily be a function of *r*. You can therefore finally determine *f(r)* from (18a).

The sufficiency of (20) is seen in the modified formula

$$\frac{1}{g'(o)}\cdot\frac{\delta r}{\delta o}=\frac{1}{h'(a)}\cdot\frac{\delta r}{\delta a} \tag{20a}$$

by viewing *r(o,a)* as a function $\bar{r}$ of $\bar{o}$ and $\bar{a}$ since for this function the following equation applies

$$\frac{\delta\bar{r}(\bar{o},\bar{a})}{\delta\bar{o}}=\frac{\delta\bar{r}(\bar{o},\bar{a})}{\delta\bar{a}} \tag{21}$$

where the general solutions are all (differentiable) functions of $\bar{o}+\bar{a}$

$$r(o,a)=\bar{r}(\bar{o}+\bar{a}) \tag{22}$$

which, when inverted to

$$\bar{o}+\bar{a}=f(r) \tag{23}$$

is identical to (18a).

10. *Specific objectivity and latent scalar additivity*. In Section 7, it was pointed out that the generality that lies in specific objectivity within a given frame of reference can be achieved if the reaction system is latently additive in one-dimensional parameters. But it can be illustrated that this condition is also necessary for specific objectivity of comparisons of objects, provided that all three sets of parameters *o*, *a* and *r* are scalar.

That a comparison between two objects O_h and O_i can be made specifically objectively means first and foremost that, from their reactions R_{hj} and R_{ij} on a random agent A_j, it is possible to derive a statement $U\{R_{hj}, R_{ij}\}$, which is independent of A_j but dependent on O_h and O_i. Since objects, agents and reactions are fully characterised by their parameters, this requires the existence of a statement about r_{hj} and r_{ij} – i. e., a function of them – that only depends on O_h and O_i. Objectivity therefore requires that there are two functions *u* and *v*, each consisting of two variables, for which

$$u(r_{hi},r_{ij})=v(o_h,o_i) \tag{24}$$

Using the terminology of (11), we can write

$$r_{hj}=r(o_h,a_j) \tag{25}$$

so that (24) becomes

$$u(r(o_h,a_j),r(o_i,a_j))=v(o_h,o_i) \tag{24a}$$

Both formulas can be used as required.

The condition for specific objectivity set up here applies regardless of the dimensionality of the three sets of parameters, but in the following we will – in continuation of the previous observations – limit ourselves to reference systems in which the parameters for objects, agents and reactions are scalar.

Furthermore, for the analysis of what (24) implies, it will to some degree be necessary to specialise the class of comparisons sought to be discovered. Here we limit this class by requiring that the three functions *r*(*x*, y), *u*(*x*, y) and *v*(*x*, y) in the studied areas for *o* and *a* have continuous partial derivatives of the first order.

Under this condition, it is possible to differentiate (24) with regard to each of the three variables o_h, o_i and a_j. According to the chain rule, this gives us

$$\frac{\delta r_{hj}}{\delta o_h}\cdot\frac{\delta u}{\delta r_{hj}}=\frac{\delta v}{\delta o_h}, \quad \frac{\delta r_{ij}}{\delta o_i}\cdot\frac{\delta u}{\delta r_{ij}}=\frac{\delta v}{\delta o_i}, \quad \frac{\delta r_{hj}}{\delta a_j}\cdot\frac{\delta u}{\delta r_{hj}}+\frac{\delta r_{ij}}{\delta a_j}\cdot\frac{\delta u}{\delta r_{ij}}=0 \tag{26}$$

where, by means of the two earlier equations, it becomes possible to eliminate the differential quotients of u in the laterr equation:

$$\frac{\delta r_{hj}}{\delta a_j}\cdot\left(\frac{\delta r_{hj}}{\delta o_h}\right)^{-1}\cdot\frac{\delta v}{\delta o_h}+\frac{\delta r_{ij}}{\delta a_j}\cdot\left(\frac{\delta r_{ij}}{\delta o_i}\right)^{-1}\cdot\frac{\delta v}{\delta o_i}=0 \tag{27}$$

Since this relationships must be valid for all o_h, o_i and a_j, we can in the first instance keep a_j constant, for instance = a_o. The coefficient for, for example, $\partial v/\partial o_h$ will thereby only be dependent on o_h, and we are free to call it $1/g'(o_h)$. Used in both components on the left side of (27), this specialisation shows that $v(o_h, o_i)$ must satisfy a partial differential equation of the form

$$\frac{1}{g'(o_h)}\cdot\frac{\delta v}{\delta o_h}+\frac{1}{g'(o_i)}\cdot\frac{\delta v}{\delta o_i}=0 \tag{28}$$

and using the same reasoning as in the conclusion of Section 8, it follows that v must be a function of the difference between

$$\overline{o}_h = g(o_h) \quad \text{and} \quad \overline{o}_i = g(o_i) \tag{29}$$

that is, of the form

$$v(o_h, o_i) = \overline{v}(\overline{o}_h - \overline{o}_i) \tag{30}$$

The function v is thus latently subtractive.

In the second instance, we let a_j vary freely in (27) but eliminate the differential quotients of v by means of (28). Thereby we get

$$\frac{\delta r_{hj}}{\delta a_j}\cdot\left(\frac{\delta r_{hj}}{\delta o_h}\right)^{-1}\cdot\frac{1}{g'(o_h)}=\frac{\delta r_{ij}}{\delta a_j}\cdot\left(\frac{\delta r_{ij}}{\delta o_i}\right)^{-1}\cdot\frac{1}{g'(o_i)} \tag{31}$$

But since the left side is independent of o_i and the right side of o_h, each side must be independent of the o in question, i.e. only dependent on a_j. We can therefore put

$$\frac{\delta r_{hj}}{\delta a_j}\cdot\left(\frac{\delta r_{hj}}{\delta o_i}\right)^{-1}\cdot\frac{1}{g'(o_i)}=h'(a_j) \tag{32}$$

which can be rearranged into

$$\frac{1}{g'(o_i)}\cdot\frac{\delta r(o_i, a_j)}{\delta o_i}=\frac{1}{h'(a_j)}\cdot\frac{\delta r(o_i, a_j)}{\delta a_j} \tag{33}$$

and from this it follows that *the function r* is latently additive, i.e. of the form

$$r(o_i, a_j) = \overline{r}(\overline{o}_i + \overline{a}_j) \tag{34}$$

in which, apart from (29), we have put

$$\overline{a}_j = h(a_j) \tag{35}$$

Combining this result with the conclusion of Section 9, we have illustrated one of the main theorems of the theory of specific objectivity:

If the parameters for objects, agents and reactions are real numbers, it is a necessary and sufficient condition for specifically objective pairwise comparisons of the objects that the reaction parameter is a latent additive function of the object and agent parameter.

To this may be added that the definition of the concept “comparison of two objects” can be extended to include comparison between several objects and that the condition for its specific objectivity is also the latent additivity of the reaction function.

Finally, it may be mentioned that, due to the fact that objects and agents appear completely symmetrically, *the latent additivity is also necessary and sufficient for specifically objective comparisons between agents*. The two kinds of objectivities go together.

11. *Production as determined by capital and job*. Since I have not had the opportunity to test the following on adequate data, it must not be taken too seriously, at least not as yet. Rather, it is a sample of what it may look like when you try to move latent scalar additivity into economics.

Production as function of capital and job

$$P = F(K, A) \tag{36}$$

is often specialised to a Cobb-Douglas function

$$P = C \cdot K^{\alpha} \cdot A^{1-\alpha} \qquad 0 < \alpha < 1, \quad \text{c constant} \tag{37}$$

On the surface, it does not appear to be multiplicative, but one could say that if the exponent α was known, then

$$K^{'} = K^{\alpha}, \quad A^{'} = A^{1-\alpha} \tag{38}$$

would have expressed capital and job in a new metric in which P is multiplicative.

Using the terminology from the previous sections, you could also say that the system (37) is latently additive and that the transformations into additivity are

$$\bar{P} = \log P, \ \ \bar{K} = \alpha \log K, \ \ \bar{A} = (1-\alpha)\log A \tag{39}$$

Naturally, if adequate data are available, it is quite easy to estimate α from them, that is, if the model fits well enough. But if possible, the question of “in which metric P, K and A should be measured in order to bring the correlation between P, K and A to expression into an additive form” could also be left open.

This way of presenting the problem would pose questions about the existence of three functions f, g and h, for which

$$f(P) = g(K) + h(A) \tag{40}$$

and these functions would have to be determined empirically as indicated in Section 9.

Of course, the way of presenting the problem may be modified, for example, as inspired by Cobb-Douglas, by entering the ratio $L = K/A$ and A as the variables that determine P. But to the actual idea, this is but a detail.

I see a main difficulty in the acquisition of adequate data for which the two – or for that matter, more – determining variables that you have fastened on should vary freely in relation to each other, but which often, for example when a single firm is studied over a number of years, accompany each other. But if it is overcome, for example by including more, differently

Table 1. 2672 students in Danish Public School distributed according to the socio-economic status of the father and the “time perspective” of the student. The table shows the percentage of students attending the 1st grade of the secondary school (“1. real” in Danish) after the eighth grade of the primary school.

Father’s	Time perspective				
social status	a	b	c	d	Total
I-II (highest)	85.1 %	78.2 %	69.8 %	54.1 %	75.1%
	n=134	n=120	n = 86	n = 61	n= 401
III	62.8 %	58.2 %	45.6 %	40.0 %	51.8 %
	n= 247	n= 256	n= 285	n= 218	n= 1006
IV	50.0 %	47.5 %	39.5 %	28.4 %	38.8 %
	n= 134	n= 139	n= 157	n= 274	n= 704
V (lowest)	43.6 %	40.7 %	25.9 %	13.1 %	28.6 %
	n= 101	n= 130	n= 162	n= 168	n= 561
Total	61.7 %	56.1 %	42.6 %	30.5 %	47.0 %
	n= 616	n= 645	n= 690	n= 721	n= 2672

sized firms that manufacture the same product, you will, when latent scalar additivity is present, get objectivity into the bargain, which should be utilisable like that of physical laws (cf. section 1), in so far as the frame of reference can be made sufficiently comprehensive.

12. *Latent additivity and probabilistic models*. When, in the previous sections, multiplicative and additive ratios, for example (3), (5) and (6), were discussed as well as differential equations, then they are, strictly speaking, only valid when the given data, the values of the reaction function *r(o, a)* are not noticeably burdened with minors errors or other random variations.

The mathematical apparatus used for the treatment of such variations is, as you will be aware, the calculation of probability. You then face the task of embedding the latently additive structures in *probability models*, where the basic principle, the specific objectivity of the comparisons that are to be made, is discovered.

Sections 13 and 14 illustrate how at times it is possible to work towards such a model.

13. *Latent additivity in percentages organised in a 4 X 4 table*. Approximately 100 years ago in his famous studies on suicide, the French sociologist Durkheim used a peculiar technique when describing tables demonstrating how the suicide frequency varies with two different social factors.

The idea in his method can be illustrated through Table 1, which is an extract of the material in Table 5.4 in Bent Bøgh Andersen (1972). Here "the time perspective" denotes the individual pupil's attitude to planning of the future, as measured by a questionnaire.

It is apparent that the percentages in each row and each column are monotonously decreasing so that the traditional "χ^2 test for independence" is without interest. It can be tempting with Durkheim to read the percentages of the table slantwise and still find systematic progress in the figures. Details in this way of viewing the table are described by the author of the report (p. 63), who has rescued Durkheim's technique from near oblivion.

A more systematic analysis technique is seemingly needed in order to arrive at a clear description of the structure of the table. What we will do then is examine whether a latently additive structure can be perceived behind the systematic features.

Obviously, since percentages are locked between 0 and 100, they cannot form an additive system themselves, but must be transformed so that, in principle, the figures achieve free mobility between – ∞ and + ∞. This is achievable through a so-called *logistic transformation*[2]

$$\lg(p) = \log\frac{p}{100-p} \tag{41}$$

where p designates a given percentage. By applying it to the percentages p_{ij} of Table 1 where i is the father's status and j the time perspective, you get the contents, $l_{ij} = log(p_{ij})$, of Table 2.

Table 2. Analysis of the percentages in Table 1: l_{ij} their average and estimated addends.

$i \backslash j$	a	b	c	d	$l_{i.}$	s_i
I-II	+0,756	+0,558	+0,284	+0,072	+0,418	+0,443
III	+0,226	+0,144	—0,076	—0,177	+0,029	+0,054
IV	0,000	—0,043	—0,186	—0,400	—0,157	—0,132
V	—0,113	—0,162	—0,456	—0,822	—0,388	—0,363
$l_{.j}$	+0,217	+0,124	—0,109	—0,332	—0,025	
t_j	+0,242	+0,149	—0,084	—0,307		

Table 3. Calculations for control of the logistic additivity of the material in table 1.

$i \backslash j$	a	b	c	d
I-II	+0,660	+0,569	+0,334	+0,111
III	+0,271	+0,178	—0,055	—0,278
IV	+0,085	—0,008	—0,241	—0,464
V	—0,146	—0,239	—0,472	—0,695

Table 4. Comparison of observed percentages and percentages calculated from $\bar{l}_{ij}$.

	a		b		c		d	
	obs.	ber.	obs.	ber.	obs.	ber.	obs.	ber.
I-II	85,1	82,1	78,2	78,7	65,8	68,9	54,1	56,4
III	62,8	65,0	58,2	60,1	45,6	46,8	40,0	34,6
IV	50,0	55,0	47,5	49,5	39,5	36,4	28,4	25,6
V	43,6	41,7	40,7	36,5	25,9	25,2	13,1	17,0

In order to see whether this simple transformation has succeeded in bringing forth the additivity, you can build on the technique outlined by the forms (13a), (15) and (16) in section 8 and here calculate the average of each row ($l_{i.}$) and each column ($l_{.j}$), as well as the total average ($l_{..}$). If theoretically you should have the relationship

[2] There are also other possibilities, but this is in several respects the simplest.

$$l_{ij} = c + s_i + t_j \tag{42}$$

- where s_i and l_j are constrained by setting their average to 0 – we would have to have

$$c \approx l_{\bullet\bullet}, \quad s_i \approx l_{i\bullet} - l_{\bullet\bullet}, \quad t_j = l_{\bullet j} - l_{\bullet\bullet} \tag{43}$$

in which the symbol $\approx$ indicates that the right side estimates the parameter on the left side.

For control of whether and how well the model fits in the present case, the estimates (43) can be inserted instead of the parameters c, s_i and t_j in (42). Thereby we get the "calculated values"

$$\bar{l}_{ij} = l_{i\bullet} + l_{\bullet j} - l_{\bullet\bullet} \tag{44}$$

in Table 3 to compare with the original l_{ij} in table 2, for example by means of a diagram with the l_{ij}s as ordinates against the corresponding $\bar{l}_{ij}$ as abscises. The result is shown in Figure 1, in which the points wind tightly around the identity line that they were supposed to lie on if the l_{ij}s could be presented precisely by form (42).

Table 4 shows how abundantly well the percentages back-calculated from $\bar{l}_{ij}$ conform to the observed percentages p_{ij}.

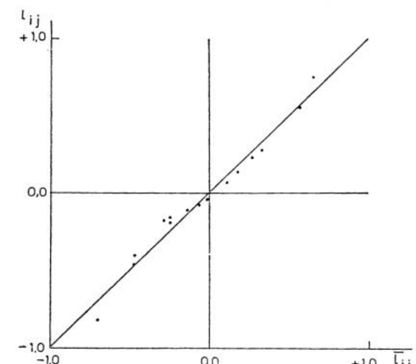

Fig. 1. Model control: l_{ij} drawn up against $\bar{l}_{ij}$

14. *Setting up an additive probability model*. A precise presentation is, of course, not possible. Percentages like p_{ij}, must at best be presumed to be subject to random variations in accordance with the binomial law with some parameter z_{ij}. Since n_{ij} is the number of pupils characterised by the combination (i, j) and a_{ij} denotes the number who went to the 1st year in secondary school, the probability of this exact number must be

$$p\{a_{ij}\} = \binom{n_{ij}}{a_{ij}} z_{ij}^{a_{ij}} (1 - z_{ij})^{n_{ij} - a_{ij}} \tag{45}$$

Since the expected value of a_{ij} in this distribution is

$$E\{a_{ij}\} = n_{ij} z_{ij} \tag{46}$$

$a_{ij} / n_{ij} = p_{ij} / 100$ could be taken as an estimate of z_{ij}. Therefore l_{ij} should also be an estimate of the logistic transformation of z_{ij}.

What the analysis in Section 13 has shown is then that there is a good chance that

$$\lg(z_{ij}) = \log \frac{z_{ij}}{1 - z_{ij}} = c + s_i + t_j \tag{47}$$

Solved with regard to z_{ij}, this relationship states that

$$z_{ij} = \frac{\exp(c + s_i + t_j)}{1 + \exp(c + s_i + t_j)} \tag{48}$$

which with

$$S_i = \exp(c + s_i)\,,\;\; T_j = \exp(t_j) \tag{49}$$

simplifies to

$$z_{ij} = \frac{S_i T_j}{1 + S_i T_j}\,, \quad 1 - z_{ij} = \frac{1}{1 + S_i T_j} \tag{50}$$

This model is as much the same as the one that has been widely used in recent years for analysis of individuals' sequences of responses to a number of questions, each with two possible answers (see, for example, chapters 12 and 13 in Ulf Christiansen and Jon Stene (1969), henceforth referred to as GR's Textbook). However, here it is used for the study of subpopulations.

The frame of reference from Sections 8 and 10 with objects, agents and reactions also applies here: the objects can be the time perspectives, which are exposed to the father's status as agents resulting in specific probabilities that pupils in the 8th school year enter the 1st year in secondary school. To this is added the assumption that *all pupils in the* (i,j) *group have the same probability* z_{ij} *of ending up in the 1st year in secondary school*. In this case, this probability is determined by the two parameters S_i and T_j, i. e., through the relationship (50), the exponential version of which (48) shows that *this reaction is latently scalarly additive*.

Finally, since the random factors in the pupils' positions are presumed to be mutually irrelevant, which is formalised as "stochastic independence", the binomial distribution (45) follows, which, with the terminology introduced in (50), takes the form

$$p\{a_{ij}\} = \binom{n_{ij}}{a_{ij}} \cdot \frac{S_i^{a_{ij}} \cdot T_j^{a_{ij}}}{(1 + S_i T_j)^{n_{ij}}} \tag{51}$$

This kind of application of so-called "**measurement models**" is dealt with in GR's textbook under the term "distribution analysis". The model (51) is only mentioned in passing, but parts of its theory have been developed by Poul Chr. Pedersen (1971).

15. *Separation of parameters and specifically objective estimation*. The calculations in Section 13 would have led to a specifically objective determination of the parameters c, s_i and t_j if (48) had been a presentation of the actual observed relative frequencies. But since this is not the case, the question remains whether it is possible to estimate the parameters with specific objectivity. This will now be examined. We will restrict ourselves to the comparison of two time perspectives j and k based on an arbitrary status i.

Since the groups (i,j) and (i,k) consist of different pupils, we have the courage to assume stochastic independence between a_{ij} and a_{ik}. According to (51), we then have

$$p\{a_{ij}, a_{ik}\} = p\{a_{ij}\}\, p\{a_{ik}\} = \binom{n_{ij}}{a_{ij}}\binom{n_{ik}}{a_{ik}} \frac{S_i^{a_{ij}+a_{ik}} T_j^{a_{ij}} T_k^{a_{ik}}}{(1 + S_i T_j)^{n_{ij}} (1 + S_i T_k)^{n_{ik}}} \tag{52}$$

In this expression, S_i appears to the power a_{ij} + a_{ik}, and the probability for a specific value r of this sum is found by writing down the probability $p\{a_{ij}, a_{ik}\}$ for every single possible value pair (a_{ij}, a_{ik}) with this sum and adding them together. Thereby, we get

$$p\{r\} = \frac{f_r(T_j, T_k)\, S_i^r}{(1 + S_i T_j)^{n_{ij}} (1 + S_i T_k)^{n_{ik}}} \tag{53}$$

in which the polynomial

$$f_r(T_j, T_k) = \sum_{a_{ij}+a_{ik}=r} \binom{n_{ij}}{a_{ij}}\binom{n_{ik}}{a_{ik}} T_j^{a_{ij}} T_k^{a_{ik}} \tag{54}$$

is homogeneously of degree r in T_j and T_k.

If the probability (53) is divided by $p\{a_{ij}, a_{ik}\}$ given in (52), you get the conditional probability for exactly those two quantities a_{ij} and a_{ik}, given that their sum is r:

$$p\ a_{ij}, a_{ik} \mid a_{ij} + a_{ik} = r\ = \sum_{a_{ij}+a_{ik}=r} \binom{n_{ij}}{a_{ij}} \binom{n_{ik}}{a_{ik}} \cdot \frac{T_j^{a_{ij}} T_k^{a_{ik}}}{f_r(T_j, T_k)} \tag{55}$$

Here it can be seen that this procedure eliminates S_i so that the probability (55) does not depend on other parameters than the ratio between T_j and T_k.

This ratio can thus be estimated based on any i, and all of these estimates (in this case 4) must be statistically compatible.

Whether they are statistically compatible can in specific cases be tested by comparing the individual pairs (a_{ij}, a_{ik}) with the total (a_{oj}, a_{ok}). However, the formulas required for this, as well as the extended system of formulas that simultaneously involves all (16) pairs (i, j), will not be treated here.

At this point, the main result must be mentioned. The first part is a generalisation of (55):

Any set of Ts can be estimated and evaluated independently of both the other Ts and all Ss and vice versa, in so far as the model (51) together with stochastic independence of the observed counts is correct.

The model hypothesis can be tested independently of all the parameters.

For the *stochastic model* mentioned here, the same applies as was emphasised for the *deterministic models* in Section 7. T*he only unknown factor in the reference system is the parameters and comparisons between objects as well as agents (cf. form (55)) can be made based on what is known* – i.e. the observed data (a_{ij}, given n_{ij}) – *unaffected by everything that is not known within the reference system. The statistical statements about the parameters that are based on this can therefore be termed "specifically objective".*

An obvious question is then which models have this remarkable quality. Mathematically, the possibilities are extremely limited. Keeping to the present situation ***with only two possible results in each instance and stochastic independence between the outcomes of the instances, the probability model given in (45) and (50) is*** – except for trivial transformations – ***the only model that allows specifically objective separation of the object and agent parameters*.**

We refer to chapter 13 in G.R.'s Textbook for the proof.

16. *Orientation towards processes.* The theory for specific objectivity and latent scalar additivity developed in Sections 1-7 and 8-10 as well as the two applications in Sections 11 and 13-15 only treat stationary systems where all reactions are determined by two fixed sets of parameters.

Through the next two examples, we will approach the problems connected to changes in a system. The stochastic problems are, however, far deeper in such areas than in the sociological example. Therefore, I must limit myself to suggestions on this occasion.

In the treatment of two data sets, we will disregard these problems and restrict ourselves to "the broad outlines" – whereby they will become examples of what I have called "numerical statistics" in a different context. Hereby, structures appear that must be implemented in future studies into the "stochastic processes" that may have generated the data.

17. *Preliminary analysis of a number of wage development curves*. From the publication *Statistikken* by *Arbejdsgiverforeningen* (the Employers Association) from the years 1953-69, P. Toft-Nielsen and Steffen Møller have extracted the hourly wages per year in 9 large areas or "industries" and made the material available to me. Sorted according to the hourly wages in 1969, it is reproduced in Table 5[3], and illustrated in Figure 2 for four characteristic cases. The remaining cases proceed according to the pattern in the three curves, while the direction of the fourth curve (F) cuts across all the other 8. These eight curves are at different levels and with different slopes, but even though some of them can be practically identical, they do not *cut* each other. The singular F cuts across several of the 8 curves, but it has the typical progression – in the beginning a weak increase, which is gradually replaced by a stronger and stronger increase that brings about a strongly curved sequence with the concavity facing upwards – in common with the other 8, only the slope is weaker.

Here it seems there is cause to look for a common structure for all nine industries, possibly a latent scalar additivity. This possibility is tested – however, in reality only for the sake of completeness – using the method indicated in Section 9. The result (not demonstrated here) was completely negative.

[3] By accident, *J*, the total over all nine industries, was treated as another industry. It is apparent in Figure 3 below, that it did not play a practical role in the following analysis.

Table 5. Average hourly wages $x_i(t)$ for 9 industries for the years

Industries	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969
E Paper and Graphics	404	428	451	502	524	545	609	650	734	784	854	931	1046	1175	1277	1426	1558
J Production companies total (a)	390	408	423	457	479	498	541	579	659	719	776	839	942	1059	1156	1277	1411
I Iron and Metal	418	422	440	473	498	518	550	601	676	747	803	864	958	1079	1164	1261	1387
D Wood and Furniture	394	407	421	453	476	485	547	585	651	715	768	836	930	1035	1117	1253	1373
A Food, Drink and Tobacco	373	392	405	438	457	473	518	542	637	683	733	789	887	993	1085	1203	1339
G Chemical	340	357	372	403	425	441	481	510	585	619	668	723	829	935	1023	1158	1298
F Leather and Leather goods	349	373	387	422	438	454	490	522	611	647	700	748	835	950	1048	1173	1287
B Textile	407	416	437	473	490	504	558	567	620	674	715	763	855	950	1018	1153	1227
C Clothes and Footwear	318	331	341	373	390	405	429	458	516	558	611	665	745	840	918	1031	1137
	307	319	333	360	380	391	435	455	515	569	617	671	756	845	918	1029	1122

Note: (a) Cf. the text.

This was, though, exactly what was to be expected since the wages in the successive years are not a fixed system where each year's hourly wages is determined regardless of the level in the previous years. It is a system in motion: through collective bargaining and wage drift, each year's hourly wages in an industry emerge from the hourly wages of the previous years.

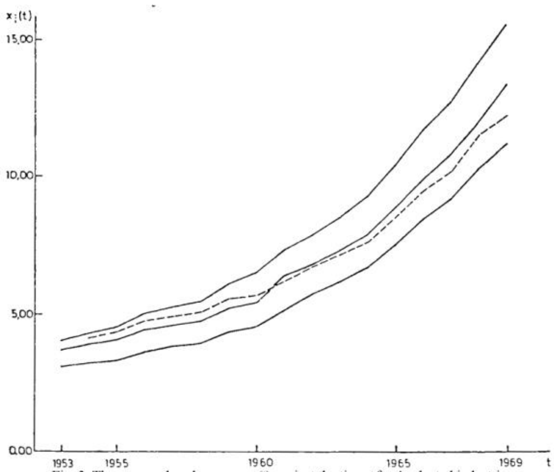

Fig. 2. The average hourly wages $x_i(t)$ against the time t for 4 selected industries.

As a first and possibly applicable approximation, we will examine whether the wage increase between two times t_1 and t_2 for the individual industry (no. i) can be formalised as a continuous process, the direction of which at a given time t is determined by three factors: the hourly wages of the industry at time t, the current financial conditions common to all the industries, and the special conditions that apply to the industry in question which are considered constant over the term of years.

Designating the hourly wages of the industry at the time t, $x_1(t)$, the speed of the increase $x_1'(t)$ will depend on $x_1(t)$ itself, on a "general economical development function" $f'(t)$ for all the industries, and on a constant parameter b_i, which is particular to the industry in question.

If there is latent scalar additivity or, in this case more conveniently, latent scalar multiplicativity in this system, there must be two such functions: first, f of the reaction $x_1'(t)$, and second, h of the agent $x_1(t)$ so that[4]

$$f\left(x_i'(t)\right) = b_i g'(t) h\left(x_i(t)\right) \tag{56}$$

Determining the unknowns – the functions f, g and h and the constants b_i and $g'(t)$ – directly from $x_1'(t)$ and $x_1(t)$ *as exactly given* would perhaps be theoretically possible, but since we would then go up to differential quotients of both 3rd and 4th order, it will be unworkable when the data in question are burdened with what must be considered measurement errors and other random fluctuations.

At present, we will instead attempt the very simple assumption

$$f\left(x_i'(t)\right) = x_i'(t), \quad h\left(x_i(t)\right) = x_i(t) \tag{57}$$

[4] there is no reason to transform b_i and $g'(t)$

that is, use as our starting point the equation

$$x_i^t(t) = b_i g^{'}(t) x_i(t) \tag{58}$$

which is integrated into

$$\log x_i(t) = a_i + b_i g(t) \tag{59}$$

in which a_i is an integration constant.

Testing this model simultaneously with an empirical determination of the function $g(t)$ and the two sets of constants a_i and b_i is a relatively simple matter: if the average of

$$y_i(t) = \log x_i(t) \tag{60}$$

is calculated over the industries, we get

$$y_{\bullet}(t) = a_{\bullet} + b_{\bullet} g(t) \tag{61}$$

If a $g(t)$ exists, we can as such simply take $y.(t)$ (or a linear transformation of it) which, when inserted into (59), gives

$$y_i(t) = a_i^{'} + b_i^{'} y_{\bullet}(t), \;\; b_i^{'} = b_i / b_{\bullet}, \;\; a_i^{'} = a_i - a_{\bullet} b_i^{'} \tag{62}$$

This equation states that if the model (59) holds and we draw a diagram for each industry with the successive values of $y_i(t)$ for t = 1953, . . . , 1969 as ordinates against the corresponding values of $y.(t)$ as abscissas, then the points must lie on a straight line with the slope of b_i'. We can choose as origin, for example, the ordinate on the line with the abscissa $\bar{y}_{\bullet}$ = the average of $y.(t)$ over t. We therefore set

$$g(t) = y_{\bullet}(t) - \bar{y}_{\bullet} \tag{63}$$

In practice, we will, of course, only get a sort of estimate of a_is, b_is and $g(t)$, and the control can at best only provide points that lie more or less closely around straight lines.

Table 6 shows the logarithms for the hourly wages as well as their average over i, and Figure 3 shows the control diagrams with parallel staggered ordinate zeros for the different industries.

Table 6. The logarithms for the hourly wages $y_i(t)$ = log $x_i(t)$ in Table 5.
Average $y.(t)$ of $y_i(t)$ over i as well as slope b_i and intercept a_i of the lines in Fig. 3.

i \ t	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	$a_i = \bar{y}_i$	b_i
E	2,606	2,631	2,654	2,701	2,719	2,736	2,785	2,813	2,866	2,894	2,932	2,969	3,020	3,070	3,110	3,154	3,192	2,862	1,024
J	2,591	2,611	2,626	2,660	2,680	2,697	2,733	2,763	2,819	2,857	2,890	2,924	2,974	3,025	3,063	3,106	3,149	2,820	1,020
I	2,621	2,625	2,643	2,675	2,697	2,714	2,740	2,779	2,830	2,873	2,905	2,937	2,981	3,033	3,066	3,100	3,142	2,832	0,970
D	2,595	2,610	2,624	2,656	2,678	2,686	2,738	2,767	2,814	2,854	2,885	2,922	2,968	3,015	3,048	3,098	3,138	2,815	0,992
H	2,572	2,593	2,607	2,641	2,660	2,675	2,714	2,734	2,804	2,834	2,865	2,897	2,948	2,997	3,035	3,080	3,127	2,797	1,000
A	2,531	2,553	2,571	2,605	2,628	2,644	2,682	2,708	2,767	2,792	2,825	2,859	2,919	2,971	3,010	3,063	3.114	2,760	1,044
G	2,543	2,572	2,588	2,625	2,641	2,657	2,690	2,718	2,786	2,811	2,845	2,874	2,922	2,978	3,020	3,069	3,110	2,775	1,016
F	2,610	2,619	2,640	2,675	2,690	2,702	2,747	2,754	2,792	2,829	2,854	2,883	2,932	2,978	3,008	3,062	3,085	2,802	0,870
B	2,502	2,520	2,533	2,572	2,391	2,607	2,632	2,661	2,713	2,747	2,786	2,823	2,872	2,924	2,963	3,013	3,056	2,725	0,994
C	2,487	2,504	2,522	2,556	2,580	2,592	2,638	2,658	2,712	2.755	2,790	2,827	2,879	2,927	2,963	3,012	3,046	2,715	1,024
$y.(t)$	2,565	2,583	2,601	2,635	2,656	2,671	2,710	2,736	2,790	2,824	2.858	2,891	2,941	2,991	3,029	3,076	3,116	$\bar{y}.$=2,804	
$g(t)$	–0,239	–0,221	–0,203	–0,169	–0,148	–0,133	–0,094	–0,068	–0,014	0,020	0,054	0,087	0,137	0,187	0,225	0,272	0,312		

We can see that the points cling fairly closely round the respective straight lines so that the model (59), and thereby also (58), must be said to offer a satisfactory representation of the present data.

It is noted that, as expected, the slope for the industry F is a great deal smaller than for the other eight, but also that differences of some importance between these slopes can be seen[5].

The estimate of the function $g(t)$ is given as the bottom line in Table 6, in which the estimates of positions a_i and slopes b_i are found as the two columns furthest to the right.

For the sake of completeness, the discovered function is drawn up in Figure 4, from which it can be read that in the first five years, the sequence of $g(t)$ corresponds to an annual increase in wages of approximately 5% while in the last 5 years, it was approximately 11%, for F somewhat lower and for A a little higher.

[5] Under additional assumptions on the random variations, you can, of course, carry out a regular statistical analysis, which, for the present purpose, I have declined to do. A possible means for this purpose is given by C. R. Rao (1958).

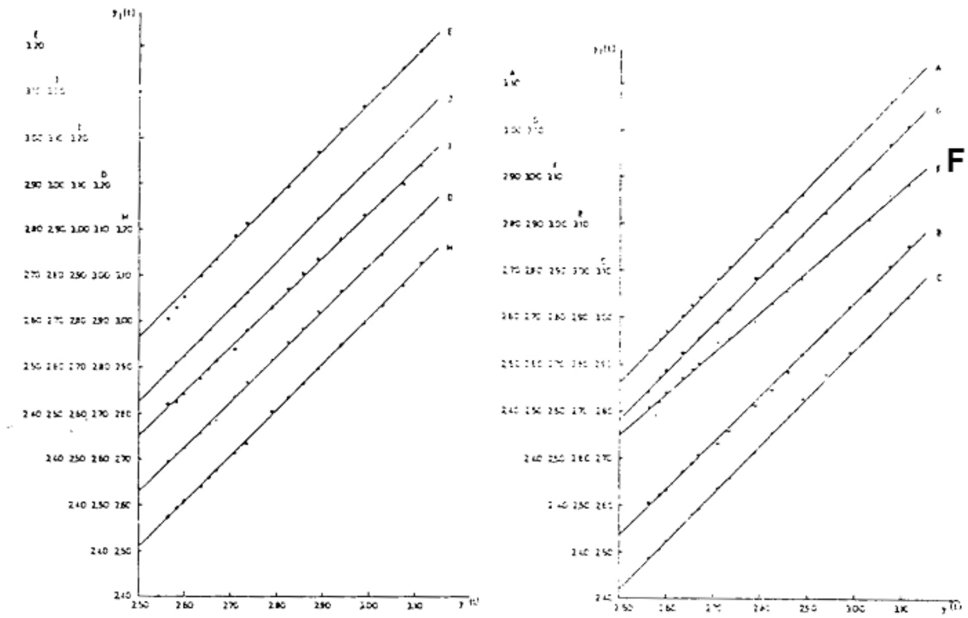

Fig. 3. The logarithms for the annual wages per company $y_i(t)$ against their average $y.(t)$.

18. *Objective evaluation of the process parameters*. With the discovered results, it is completely clear, including technically, why the first test broke completely down: $x_i(t)$ as determined by time and industry contains *a scalar parameter $g(t)$ per time, but a two-dimensional parameter* (a_i, b_i) *per industry,* while the theory in sections 1-10 presupposes that there are only one-dimensional parameters.

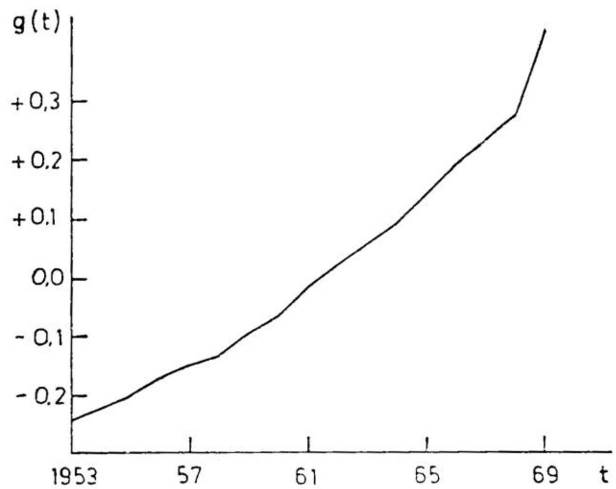

Fig. 4. The sequence of $g(t) = y.(t) - \bar{y}.$ over time.

However, viewing the wage development as a process brings out the multiplicativity, as illustrated by (58) re-expressed as

$$\frac{x_i'(t)}{x_i(t)} = \frac{d \log x_i(t)}{dt} = b_i g'(t) \tag{64}$$

when viewing log $x_i(t)$, and not $x_i(t)$ itself, as the thing that changes.

Employing this "process" point of view, we have reached the latent scalar additivity of two sets of parameters, the industry parameter b_i and the general sequence parameter $g'(t)$, which can then be evaluated specifically objectively. Conversely, the determination of a_i falls outside the developed theory of objectivity.

Incidentally, the statement (64) corresponds to the fact that in wage negotiations and wage drift, people may *talk* about actual money, however in reality, they *think in terms of* relative wage increases (cf. the concluding remark in Section 17).

19. *Structure in mortality data*. The third example drawn from demographics is about the variation of the death intensity with age for men in Denmark in the years 1906-1955, the age x (5-75 years old) for every 5th year and the calendar year t grouped in intervals of 5 years. The death intensity is, in principle, to be understood here as the number calculated per 100,000 among those who in the time span (t, t+5) turned x years old who died before their next birthday.

The basic data in Table 7 naturally shows very large variation with age, which complicates immediate comparisons between the age levels. As an attempt to compensate for it, we take the logarithms shown in Table 8; the effect is illustrated in Figure 5, which shows for each age level how log q_{xt} has changed over the course of 50 years. The sequences are fairly steady, however, for the ages up to 40 interrupted by strong peaks upwards for t = 1916, i.e. for the five-year period 1916-1920 with the two great epidemics of "the Spanish flu". But outside this period – and apparently without major lasting effects of it – we see a steady decline over the years, strongest for children and youths, still considerable from ages 40 to approximately 60, but flattening more and more for the elderly.

This observation tempts us to seek a structure, but taught by the experience with the hourly wages, we will not directly seek a latently additive structure. However, regardless of the lack of a basis such as (58), we must ask purely geometrically whether the curves in Figure 5 – similarly to the logarithmical wages of the 9 industries – can be linearly transformed onto each other.

The bottom row in Table 7 gives us the average ("unweighted") over ages for each time-interval, and with these as abscissas drawn in Figure 6 for each age level, it becomes a diagram with the values of l_{xt} = log q_{xt} as ordinates. (x = 25 and x = 35 are omitted as they almost coincide with x = 20 and x = 35 respectively.) The points for 1916 are framed by circles and are in general level with the other points, which otherwise for each age level gather around a straight line. The variations are obvious enough, and it is really not unthinkable that another structure layer could be revealed through careful examination. But the main thing is that, in any case, there is an obvious primary structure that can be expressed as the linear relation of l_{xt} to $l_{.t}$ for each age group:

$$l_{xt} \approx a_x^{'} + d_x l_{\bullet t} = a_x + d_x l_t , \quad l_t = l_{\bullet t} - \overline{l}_{\bullet\bullet} \qquad (65)$$

Table 7. The death intensities q_{xt} for men in Denmark 1906-55 (x = age; t = time span).

x \ t	1906	1911	1916	1921	1926	1931	1936	1941	1946	1951
5	285	275	299	183	168	134	136	113	86	68
10	181	160	194	127	118	113	98	88	59	42
15	248	206	265	160	163	147	108	113	73	53
20	386	405	654	323	288	256	206	209	158	122
25	404	412	638	328	249	266	224	225	170	128
30	447	462	664	328	291	268	228	223	187	145
35	528	530	647	335	323	324	281	258	211	157
40	689	609	644	434	450	401	389	335	291	221
45	938	804	756	588	562	584	525	462	455	364
50	1187	1138	1024	805	830	832	754	740	664	618
55	1707	1538	1487	1220	1208	1244	1191	1087	1053	986
60	2389	2342	2066	1937	1977	1866	1885	1710	1626	1536
65	3403	3358	3200	3059	2989	3097	2886	2633	2493	2473
70	5363	5122	4730	4755	4660	4825	4901	4375	4053	3951
75	8079	8316	7589	7794	7671	7837	7729	7044	6668	6484

Source: Befolkningsudvikling og Sundhedsforhold 1901-60 [*Population development and Health conditions 1901-60*], published by Det statistiske Departement, Statistiske Undersøgelser no. 19. Copenhagen 1966, table 23.

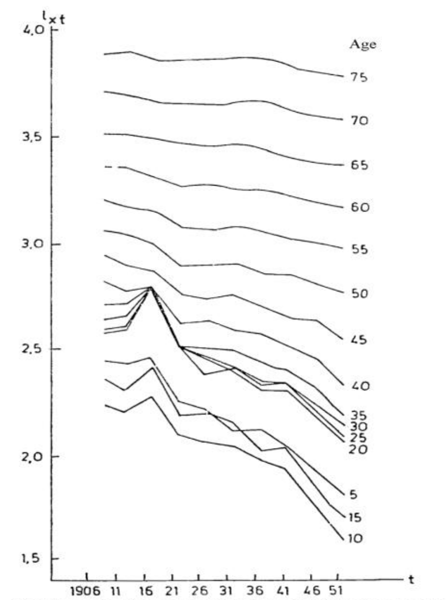

Fig 5. The sequence of the logarithm for the death intensity with time for each age level.

The slope of the straight lines d_x and the positions a_x determined as ordinates for the abscissa for $\overline{l}_{.}$ = the average over t of $l_{.t}$ is read and inserted as the two last columns in Table 7. The sequence of the time function $l_{.t}$ is shown in Figure 7; except for a peak upwards in the t = 1916 curve mentioned above, it shows a steady decline. Figure 8 shows that in broad outline, the slope d_x decreases monotonously with age, however with a plateau from ages 20 to 40; the fluctuations may be real, but they may partly be due to the uncertainties in the graphical determination of the slopes, since it is sensitive to the variations of the points around the lines, which are not quite small. This variation does not much affect the reading of the positions a_x which also, as illustrated in Figure 9, shows a steady monotonous sequence except the fall from ages 5 to 10 which, however, is quite real and already emerges clearly from Figure 5.

Table 8. Logarithms for death intensities $l_{xt} = \log(q_{xt})$.

x \ t	1906	1911	1916	1921	1926	1931	1936	1941	1946	1951	a_x	d_x
5	2,455	2,439	2,476	2,262	2,225	2,127	2,134	2,053	1,940	1,833	2,21	1,62
10	2,258	2,204	2,258	2,104	2,072	2,053	1,991	1,944	1,771	1,623	2,03	1,54
15	2,394	2,314	2,423	2,204	2,212	2,167	2,033	2,053	1,863	1,724	2,15	1,60
20	2,587	2,607	2,816	2,509	2,459	2,408	2,314	2,320	2,199	2,086	2,41	1,30
25	2,606	2,615	2,805	2,516	2,396	2,425	2,350	2,352	2,233	2,107		
30	2,650	2,665	2,822	2,516	2,464	2,428	2,358	2,348	2,272	2,161	2,47	1,30
35	2,723	2,724	2,811	2,525	2,509	2,511	2,449	2,412	2,324	2,196		
40	2,838	2,785	2,809	2,637	2,653	2,603	2,590	2,525	2,464	2,344	2,63	1,32
45	2,972	2,905	2,879	2,769	2,750	2,766	2,720	2,665	2,657	2,561	2,79	0,98
50	3,074	3,056	3,010	2,906	2,919	2,920	2,877	2,869	2,822	2,791	2,92	0,68
55	3,232	3,187	3,172	3,086	3,082	3,095	3,076	3,036	3,022	2,994	3,11	0,52
60	3,378	3,370	3,315	3,287	3,296	3,271	3,275	3,233	3,210	3,186	3,29	0,48
65	3,532	3,526	3,505	3,486	3,476	3,491	3,460	3,420	3,397	3,393	3,47	0,36
70	3,729	3,709	3,675	3,677	3,668	3,683	3,690	3,641	3,608	3,597	3,67	0,28
75	3,907	3,920	3,880	3,892	3,885	3,894	3,888	3,848	3,824	3,812	3,88	0,32
$l_{.t}$	2,956	2,935	2,978	2,825	2,804	2,789	2,747	2,715	2,641	2,561		

20. *A comment on the discovered structure*. The structure relation (65) is of quite a singular form as it states that in the period in question, the death risk for men was by and large only determined by their age x and the time t when they were that age, while what had happened before their lifetime – within hygiene, medicine, technology, social conditions etc. – in any case only played a secondary role with regard to the current condition in society at the time in question.

Is it possible that such a peculiar result could be a, however almost incredible, statistical trick that these data from Denmark in just those 50 years have played on us?

However, to this can be added that approximately 10 years ago, P. C. Matthiesen carried out a similar study of Swedish data and found a quite similar result (unpublished). Furthermore that among the mortality data from a number of countries found in the publication United Nations (1955), 19 countries that had at least 4 registration times were chosen for a preliminary study in a seminar at the Statistisk Institut (*Statistical Institute*) in the Spring of 1969 and later analysed more in depth by Peter Allerup in an exam paper: everywhere, the same structure was found.

It therefore seems that we will have to resign ourselves to the structure revealed in the Danish data – if nothing else then at least as a first step towards the formulation of a structure describing the effect of age on mortality under different local and temporary conditions that is common for many places in the world.

21. *The problem of objectivity in the case in question*. With regard to the problem of objectivity, it must first be noted that, when viewed locally, we have in (65) a situation similar to the one in (64), i.e. a one-dimensional parameter l_t per time, but a two-dimensional parameter (a_x, d_x) per age level. It is not covered by the previously developed theory, but it invites an extension of the frame of reference so that the restrictions of one-dimensionality of the parameters for objects, agents and reactions are loosened. An extension to higher dimensions does exist, but only under the condition that all 3 kinds of parameters have the *same* dimension. This is a restriction there may be cause to attempt removing.

However, just as in the wage example, we can apply the point of view that what is observed, in this case the death risk for men at a given age, is something that changes over time, and for this change in the time span (t_1, t_2), we have according to (65)

$$l_{xt_2} - l_{xt_1} \approx d_x \ \ l_{t_2} - l_{t_1} \tag{66}$$

that is, latent additivity and thereby specific objectivity.

Now the death intensities – or, if you will, their mathematical correlate – are defined as the logarithmical differential quotients of the share of the population – at a given time – who have survived given ages. Designating this share L_{xt}, we get

$$q_{xt} = \frac{\delta \log L_{xt}}{\delta x} = \frac{1}{L_{xt}} \cdot \frac{\delta L_{xt}}{\delta x} \tag{67}$$

But hereby, we have already introduced a process point of view: how the population dies out with age. What is added in (66) – or its differential counterpart $\dfrac{\delta l_{xxt}}{\delta t} = d_x \cdot \dfrac{dl_t}{dt}$ (66a) is also the application of the "process" point of view to the time sequence.

We then reach the conclusion that *if the purely static point of view, i.e. the distribution of the population on age at a given time, is replaced with the process point of view for changes of the death risk with both age and time, then in the observed case, we achieve specifically objective separation of the remaining scalar parameters.*

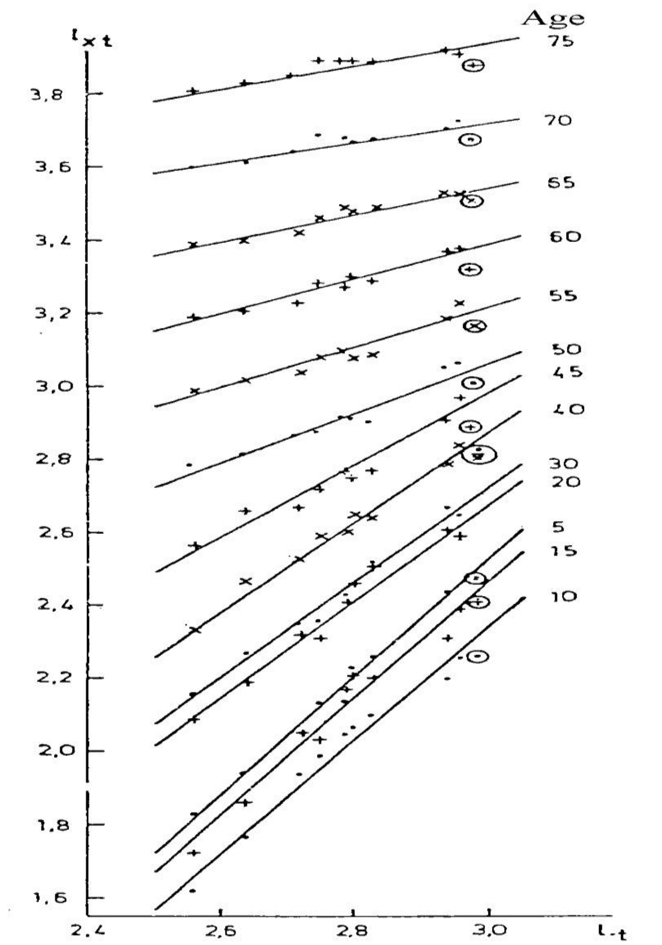

Fig. 6. The logarithms l_{xt} of the death intensities for each age group drawn against their average $l_{\cdot t}$ per time.

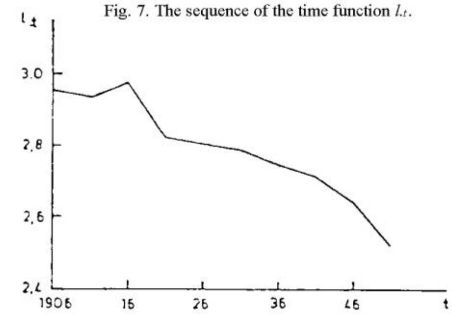

Fig. 7. The sequence of the time function $l_{\cdot t}$.

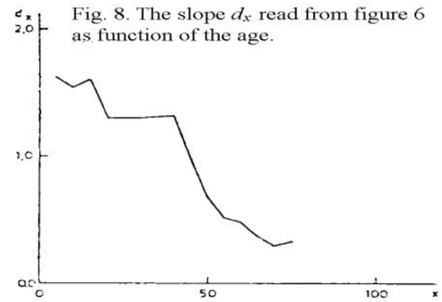

Fig. 8. The slope d_x read from figure 6 as function of the age.

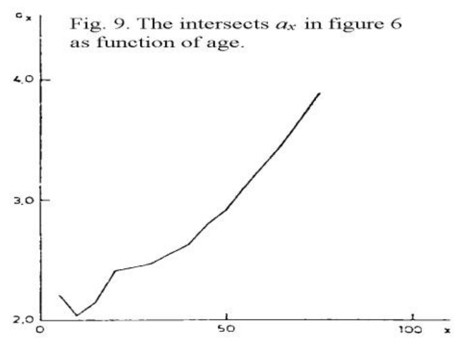

Fig. 9. The intersects a_x in figure 6 as function of age.

22. ***Specific objectivity in processes?*** The analyses of the wage development within the industrial sector in the years 1953-69 and of the changes in the death risk for men in Denmark in the years 1906-55 pointed at the possibility for achieving specific objectivity in cases where there have been changes during the observation period and where each observation is therefore based on that or the previous periods.

They pointed in particular to the importance, as a condition for achieving latent additivity, of not taking the actual wages or actual risk of mortality respectively for the reaction, but rather the changes in them.

These two cases must be said to be kinetic when reference is only to changes over time, not to the influences that brought them about. But kinetic and dynamic phenomena can be summarised under the term *processes* where agents can be actual influences as well as time and time intervals.

23. *The dynamic problem.* In order to gain insight into the dynamic problem, we return to the solid bodies that are affected by mechanical moving instruments. However, we shall take the discussion a little further.

At a given time, a body L moves in relation to Earth with a velocity V_0 that is changed to V_1 under an influence in the direction of movement by an instrument I which gives it an acceleration of $A = V_0 - V_1$.

In the previous discussion (Sections 2 and 3), we found that this acceleration is proportional to a parameter for the instrument, its "force", and vice versa proportional to a parameter for the body, its "mass".

The body, which was first in a condition where it had the velocity V_0, is brought by the influence to a new condition where it has the velocity

$$V_1 = V_0 + G\frac{F_1}{M} \tag{68}$$

where F_1 designates the force of the instrument and M the mass of the body, cf. section 3, forms (3) and (4).

But the body in its new condition can now be influenced by another (or the same) instrument with the force F_2, receive the acceleration GF_2/M and thereby be brought to a new condition with the velocity

$$V_2 = V_1 + G\cdot\frac{F_2}{M} = V_0 + G\cdot\frac{F_1+F_2}{M} \tag{69}$$

cf. equation (5a). And so forth. Through n such influences, the initial velocity V_0 is gradually changed into

$$V_n = V_0 + G\cdot\frac{F_1+\ldots+F_n}{M} \tag{70}$$

Throughout these changes, the body retains its *permanent parameter*, the mass M, while the *condition parameter*, the velocity V, goes through changing values.

24. *The frame of reference for processes*. We can now set up a frame of reference for processes in continuation of the one set up for statics in section 6.

Once again, we have objects O, agents A and reactions R, but to this is added that an object can exist in different conditions T and that the transition from one condition to another happens through a transformation that is brought about by the reaction R effected by an agent A on the object in the previous condition. The sequence can be thus schematically presented:

$$\begin{array}{ccccccc} & & A_1 & & A_2 & & A_3 \quad \ldots \\ O_0^{(1)} & : & R_1^{(1)} \to O_1^{(1)} & : & R_2^{(1)} \to O_2^{(1)} & : & \ldots \\ O_0^{(2)} & : & R_1^{(2)} \to O_1^{(2)} & : & R_2^{(2)} \to O_2^{(2)} & : & \ldots \\ \ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots\ldots & \ldots\ldots & \ldots\ldots\ldots\ldots\ldots & \ldots\ldots & \ldots \end{array} \tag{71}$$

in which the upper index of O is the identification of the object, which is constant during the whole process, while the lower index of O marks the changing conditions.

The purpose of the following analysis is to develop tools for comparison within this frame of reference: comparisons between objects with reference to how the observed kinds of processes elapse. In this, it is implied that the problem is not the description of a single sequence, for example a single TIME SERIES, say in the price development for potatoes in Denmark from 1919 to 1955, but rather the price development for many kinds of vegetables and other foods, possibly other goods. Furthermore, the comparisons are between conditions, regardless of the objects where they occur. And finally, the comparisons are between agents as they work on any conditions with any objects.

25. *Parameterisation and specific objectivity for processes*. Parameterisation encompasses agent parameters a, the permanent parameters of the objects o and their condition parameters t, as well as the parameters of the reaction r, which in any given situation are unambiguously determined by the 3 other parameters:

$$r = r(o,t,a) \tag{72}$$

Similarly to what was done in the static example in section 10, we designate here a comparison of, for example, two objects with the parameters o_1 and o_2 specifically objective if it is independent of the other parameters in the frame of reference. Since the statement must be based on what is known, i.e., the observed reactions r, the demand implies the existence of a function u of the two rs, which is dependent on the two o's that are to be compared. Therefore

$$u\;\;r\;\;o_1,t,a\;\;,r\;\;o_2,t,a\;\;\; = v(o_1,o_2) \tag{73}$$

cf. formula (24).

Still limiting ourselves to scalar parameters, it can be demonstrated that *if specific objectivity is demanded in all 3 directions, the reaction r must be latently additive in all 3 variables*.

The proof of it runs almost parallel to the one that was given in section 10.

By alternately differentiating (73) with regard to all 4 variables, we get 4 equations of which the two differential quotients of u with regard to $r_1 = r(o_1, t, a)$ and $r_2 = r(o_2, t, a)$ can be eliminated. Hereby we get

$$\frac{\delta r_1}{\delta t}\left(\frac{\delta r_1}{\delta o_1}\right)^{-1}\frac{\delta v}{\delta o_1}+\frac{\delta r_2}{\delta t}\left(\frac{\delta r_2}{\delta o_2}\right)^{-1}\frac{\delta v}{\delta o_2}=0 \tag{74}$$

and the analogous equation by differentiation with regard to a. If we give t and a special values t_0 and a_0 and introduce the designation

$$\left.\frac{\delta r(o,t,a)}{\delta t}\left(\frac{\delta r(o,t,a)}{\delta 0}\right)^{-1}\right|_{t=t_0;a=a_0} = f_1^{-1}(o) \tag{75}$$

we get

$$\frac{\delta r_1}{\delta t}\left(\frac{\delta r_1}{\delta o_1}\right)^{-1} f_1(o_1)=\frac{\delta r_2}{\delta t}\left(\frac{\delta r_2}{\delta o_2}\right)^{-1} f_1(o_2) \tag{76}$$

and the analogous equation. From this it follows that these four expressions are only dependent on t and a, and we can therefore gather the equations in

$$f_1^{-1}(o)\frac{\delta r}{\delta o}=g_1^{-1}(t,a)\frac{\delta r}{\delta t}=h_1^{-1}(t,a)\frac{\delta r}{\delta a} \tag{77}$$

In the same way, we get for the specific objectivity in the two other directions

$$f_2^{-1}(t)\frac{\delta r}{\delta t}=g_2^{-1}(o,a)\frac{\delta r}{\delta o}=h_2^{-1}(o,a)\frac{\delta r}{\delta a} \tag{78}$$

and

$$f_3^{-1}(a)\frac{\delta r}{\delta a}=g_1^{-1}(o,t)\frac{\delta r}{\delta o}=h_1^{-1}(o,t)\frac{\delta r}{\delta t} \tag{79}$$

But if the three sets of equations are to apply simultaneously, they must be reducible to one set of the form

$$f_1^{-1}(o)\frac{\delta r}{\delta o}= f_2^{-1}(t)\frac{\delta r}{\delta t}= f_3^{-1}(a)\frac{\delta r}{\delta a} \tag{80}$$

which with

$$\bar{o}=\int f_1(o)do,\quad \bar{t}=\int f_2(t)dt,\quad \bar{a}=\int f_3(a)da \qquad 81)$$

as new variables and with the designation

$$r(o,t,a)=\bar{r}(\bar{o},\bar{t},\bar{a}) \tag{82}$$

is reduced to

$$\frac{\delta\bar{r}}{\delta\bar{o}}=\frac{\delta\bar{r}}{\delta\bar{t}}=\frac{\delta\bar{r}}{\delta\bar{a}} \tag{83}$$

with a random function of $\bar{o}+\bar{t}+\bar{a}$ as the complete solution.

26. *Status and perspectives*. My purpose in this retirement lecture was to give the audience insight into the trains of thought that studies within psychology in the ‘50s and taking over the Chair in *Theoretical Statistics as a Tool within Social Sciences* provoked me to take up on a wide basis.

As far as it has been worked out, the theory is already quite comprehensive, so on this occasion, heavy cutting was necessary. This has been carried out in two directions, partly through a limitation to cases where all the parameters are one-dimensional, which leads to both results and proofs becoming relatively simple; and partly by restricting ourselves to the situation where only 2 different responses are possible in indeterminate cases.

The one-dimensionality restriction ensures that the reactions – which in the indeterminate case are the probability distributions of the two possible responses – become latently additive. And just two possible responses carry forward into as simple a distributional form (50) as possible.

If the dimensionality model is extended but it is maintained that the three kinds of parameters – for objects, agents and reaction – must have the same dimensionality[6], then the deduction of the differential equation (28) still applies when the terms used are interpreted as vectors and matrices. But from this, a multi-dimensional latent additivity only follows under much more restrictive conditions, which, by the way, cannot be said to have been completely mapped yet. However, one highly applicable sufficient condition is available.

If in the stochastic problem the range of possible responses is extended beyond 2, the above mentioned sufficient condition leads to a mathematically extremely limited, yet in practice still highly comprehensive, class of distributions, the so-called "measurement models". Furthermore, these models, which naturally encompass the simple dichotomous model (50) as a special case, can be extended to cases where the responses can be distributed over the entire real axis in the plane or in space, and thereby they give us a considerable extension of the classical statistical arsenal.

An extension in a third direction, which has indeed been predicted but which has as yet not been intensively cultivated, is beginning to make itself felt. It involves interactions between more kinds of elements than objects and agents. H. Scheiblechner (1971) recently called attention to the extension as an important tool within sociology and social psychology, and it will certainly be completely essential in analyses of financial systems.

The kind of mathematical problems that this extension causes has already crept into the treatment of the process problem in section 25 where there is a correlation between 3 kinds of parameters: one for agents and two for objects, i.e., a permanent one for the objects as such and one for their changing conditions.

This modest glimpse into the theory of processes opens up wide perspectives since all the available results from the objectivity of statics can be transferred directly onto the processes – also those where the relationship takes place between more than two kinds of elements. It will apply to both deterministic processes and stochastic processes so that the measurement models are carried into "measurement processes".

With all of this available to us, we will have an instrumentarium with which many kinds of problems in the social sciences can be formulated and handled with the same types of mathematical tools that physics has at its disposal – without it becoming a case of superficial analogies.

But why stop at social sciences. **My vision stretches to *all sciences where the subjects are comparisons that must be objective*.**

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*[**Bolding** and formatting added by Editor]*

[6] Lack of balance between the dimensions has, as yet, not been treated satisfactorily.