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Ask the Experts: Rasch vs. Factor Analysis

The "Ask the Experts" series is a new feature in *RMT* in which experts throughout the world weigh-in on a number of controversial topics. For this issue, I have selected the topic of Rasch versus factor analysis. I selected this topic because numerous Rasch enthusiasts have mentioned many journal reviewers and editors continue to confuse the methodologies and sometimes require additional, and unnecessary, data analyses. Thus, the purpose of this piece is to provide readers with authoritative insights on Rasch versus factor analysis and help Rasch advocates overcome these common objections to Rasch analyses.

The expert panel for this piece includes Karl Bang Christensen from the Department of Biostatistics at the University of Copenhagen (Denmark), George Engelhard, Jr. from the Department of Educational Studies at Emory University (USA), and Thomas Salzberger from the Department of Marketing at WU Wien (Austria).

"Rasch vs. FA" – Karl Bang Christensen

Rasch models have been confirmatory in nature since the seminal work of Georg Rasch (Rasch 1960; 1961). Thus, it is natural to consider when a Rasch analysis should be combined with a confirmatory factor analysis.

Exploratory factor analysis is a method for an entirely different situation, where no pre-specified hypothesis is tested. Furthermore, for a data set in, say, SPSS the user has to choose between seven options for 'extraction method', six options for 'rotation', and between covariance and correlation matrix. Even if a 'true model' exists there is little chance that choosing between these 84 different options yields a correct result.

Before deciding on Rasch analysis, confirmatory factor analysis, or a combination of the two, we need to consider the following: "what question do we want to answer?" We may outline different situations:

(i) We feel confident that items function well with regard to targeting, DIF and with nothing in the item content to suggest local dependence and the only unanswered question is dimensionality.

(ii) We feel less confident about the items, and want to study dimensionality along with evidence of local dependence, DIF and item fit.

(iii) In a given data set, we want to reduce a (possibly large) set of items to a small number of summary scale scores.

In situation (i) confirmatory factor analysis is adequate. Factor analysis based on polychoric correlations is likely to be at least as efficient as Rasch Analyses for disclosing multidimensionality. Larger correlations makes it more difficult to detect, but power of the tests increase with the sample size.

Situation (ii) is an example where confirmatory factor analysis alone is insufficient, mainly due to its inability to address spurious evidence (Kreiner & Christensen, 2011b). The Rasch model is the appropriate choice, possibly combined with confirmatory factor analysis.

Situation (iii) calls for Rasch analyses to be combined with exploratory and confirmatory factor analyses.

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Unidimensionality is important and should be seen as one end of a continuum. Rather than asking 'unidimensional or not?', we should ask 'at what point on the continuum does multidimensionality threaten the interpretation of item and person estimates?' (Smith, 2006, p. 206). The Rasch literature is vague about this requirement and about recommendations as to its assessment (Smith, 1996). It is unreasonable to claim unidimensionality based solely on item fit statistics. However, unidimensionality is often assumed, rather than explicitly tested.

Infit and outfit test statistics summarizing squared standardized response residuals are widely used to test fit of data to the Rasch model, even though results concerning their distribution are based on heuristic arguments known to be wrong (Kreiner & Christensen, 2011a). When most items measure one dimension item fit statistics flag remaining items as misfitting. Item fit statistics are unlikely to have any power against multidimensionality, for dimensions with equal numbers of items, but patterns in residuals can indicate multidimensionality (Smith, 2002).

Response residuals should be interpreted with caution since their distribution is not known; however, indirect evidence shows when fitting unidimensional Rasch models to data where two underlying latent variables are responsible for the correlations, typically they result as negative correlation between residuals from items in different dimensions. However, no formal test is obtained. Importantly, evidence of local dependence should not automatically be interpreted as evidence of multi-dimensionality.

Formal tests can be obtained (e.g., the Martin-Löf test which is a likelihood ratio test statistic). Using a chi-square approximation will be useful only for disclosing multidimensionality in large samples when the correlation is modest (Christensen et al., 2002). Monte Carlo approaches that yield more powerful, but also time-consuming tests (Christensen & Kreiner, 2007) are also implemented.

The 't-test approach' tests equivalence of person estimates from two subsets of items (Smith, 2002), after converting the estimates to the same metric. The original approach compared estimates generated on subsets of items to estimates derived from the complete item set. However, in this situation the estimates are not independent (Tennant & Conaghan, 2007).

When the distribution of person location estimates is approximately normal a high proportion of persons with significantly different locations can be taken as evidence against unidimensionality, but since estimates of person locations for extreme scores are biased and non-normal, a cautious approach is recommended for skewed score distributions.

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"Rasch measurement theory and factor analysis" – *George Engelhard, Jr.*

Social, behavioral and health scientists increasingly use Rasch measurement theory (RMT) to develop measures for the key constructs included in their theories of human behavior (Engelhard, in press). As the number of research publications based on RMT increases, journal editors and peer reviewers who are unfamiliar with modern measurement theory may ask questions about the relationship between RMT and factor analysis (FA).

There are a variety of ways to view the relationships among RMT and FA. My perspective is represented in Figure 1. First of all, I view measurement through the philosophical lens of invariant measurement (IM). IM has been called "specific objectivity" by Rasch, and other measurement theorists have used other labels (Engelhard, 2008). The five requirements of IM are as follows:

Person measurement:
1. The measurement of persons must be independent of the particular items that happen to be used for the measuring: *Item-invariant measurement of persons.*
2. A more able person must always have a better chance of success on an item than a less able person: *Non-crossing person response functions.*

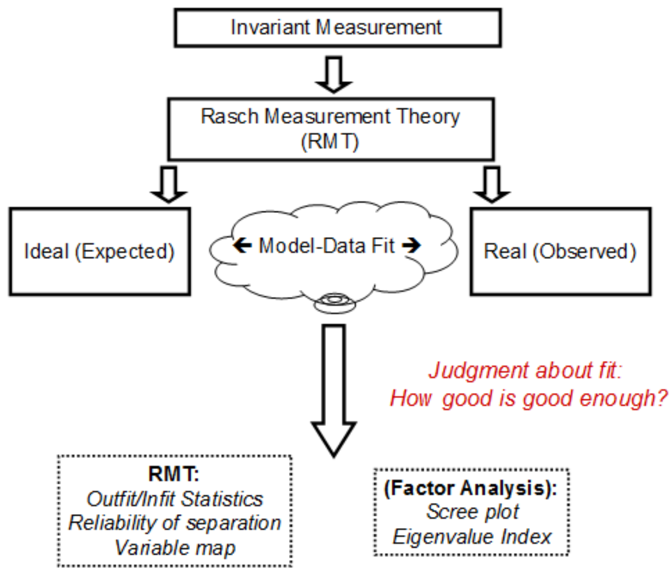
Item calibration:
3. The calibration of the items must be independent of the particular persons used for calibration: *Person-invariant calibration of test items.*
4. Any person must have a better chance of success on an easy item than on a more difficult item: *Non-crossing item response functions.*

Variable map:
5. Items and person must be simultaneously located on a single underlying latent variable: *Variable map.*

RMT can be viewed as a psychometric model that can meet the requirements of IM when there is acceptable model-data fit. In essence, RMT embodies ideal-type models that meet the requirements of IM. However, it is important to stress that with real data, IM reflects a set of hypotheses that are examined with a variety of model-data fit indices. As shown in Figure 1, I view the customary RMT indices of model-data fit (e.g., Outfit/Infit statistics, reliability of separation indices, and variable maps) as support for the inference that a particular data set has approximated the requirements of IM. Some of the analytic tools from FA can also be used to provide evidence regarding fit and unidimensionality, such as scree plots and eigenvalue-based indices Reckase (1979). Randall and Engelhard (2010) provide an illustration of using confirmatory FA and RMT to examine measurement invariance.

RMT and FA provide analytic tools for exploring model-data fit to explore hypotheses regarding invariant measurement. No single model-data fit index can detect all of the possible sources of misfit. Model-data fit is sample-dependent, and the key question in judging fit is: How good is good enough? There is no definitive statistical answer to this question, but various indices (including FA) can provide evidence to support inferences regarding invariance within a particular context.

Figure 1. Conceptual Framework



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"The Rasch model and factor analysis: Complementary or mutually exclusive?" – *Thomas Salzberger*

Striving for the same goal?

The Rasch model (RM) and factor analysis (FA) claim to serve the same purpose: measurement. This raises several questions. What is their relationship? Can we dispense with factor analysis altogether? Should Rasch analysis and factor analysis be carried out in a complementary fashion or side by side? There are no unambiguous answers to these questions; at least not if we take sociology of science into account.

RM and FA can be compared at the rather technical micro-level, which we will discuss later, or at the "philosophical" macro-level. At the latter, invariance as the defining property of the RM (Andrich 1988, 2010) is crucial. If invariance is empirically supported across samples from different subpopulations and occasions, in other words, across space and time, then measures are comparable and a uniform latent variable is a viable assumption within the established frame of reference. By contrast, if item parameter estimates fail to replicate across different samples or occasions, no common frame of reference can be established and the hypothesis of a uniform latent variable is untenable.

Multi-group FA (MG-FA) extends the idea of invariance to FA by imposing equality constraints mostly on factor loadings, item intercepts, and error variances (Meredith, 1993). This procedure has shortcomings, though. FA models do not separate respondent and item properties. Thus, factor loadings and item intercepts are sample dependent. It is therefore questionable whether truly invariant items will necessarily show invariance in MG-FA when respondent distributions and the targeting markedly differ. Furthermore, FA is associated with a series of highly problematic assumptions (see Wright 1996) with interval scale properties of item scores being probably the most serious (and generally deemed very unlikely) supposition. The point, though, is that if item scores *are* linear measures then FA *is* justified and the application of the RM *is not*. The reason for the latter is that the non-linear transformation of the raw score would be incorrect, since the raw score is already linear. Conversely, if the item scores are non-linear, the application of FA is unjustified (see Waugh and Chapman, 2005), while the RM is appropriate. This implies that the RM and FA are, strictly speaking, incompatible, mutually exclusive models. While the RM, by assessing fit, investigates whether observed person raw scores can be converted into linear person measures and observed item raw scores into linear item measures, FA requires measures as the input.

Misfit of the data to the RM implies that item scores are not even ordinal or non-linear (Salzberger, 2010), but merely numerals arbitrarily assigned to response options. Ironically, this is what proponents of Stevens' (1946, 1951) definition of measurement mistake for constituting measurement and what represents a factor analyst's only "evidence" of measurement at the item level. In other words, FA requires what it purports to provide: measures. If one rejects Stevens' definition of measurement and deems invariance a necessary requirement of measurement, there is, in fact, no point in applying FA in addition to the RM.

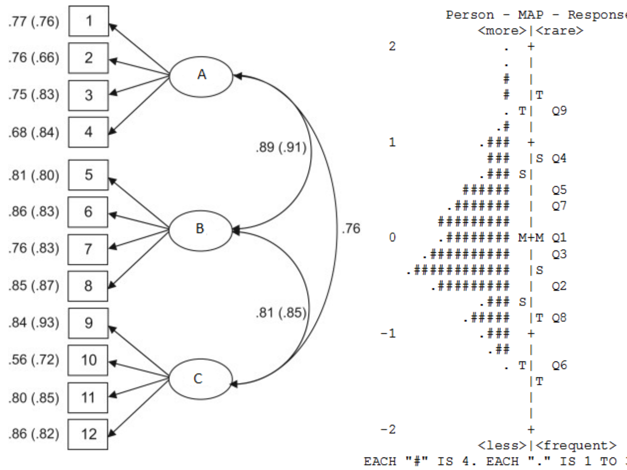

Figure 1. Sample CFA and Rasch Output

A pragmatic perspective

From a more pragmatic point of view, one might argue that even though the FA of non-linear scores is, strictly speaking, wrong, FA, specifically exploratory FA, may provide insights that inform a subsequent Rasch analysis. In a simulation study, Smith (1996) found that FA outperforms Rasch fit analysis in the assessment of unidimensionality in a two-factor model, when the correlation between the factors is small (<0.30) and the number of items per dimension balanced. By contrast, with higher correlations and uneven numbers of items, the fit statistics in the Rasch analysis are more powerful. Thus, from a technical point of view FA could be used prior to a Rasch analysis as a tool to generate hypotheses of separate unidimensional variables. Having said that, proper scale development and analysis should never be confined to a statistical procedure (even if that procedure utilizes the RM), but should be guided by a theory of the construct to be measured. It is hard to imagine how the existence of two hardly related dimensions can go unnoticed in previous qualitative work. Moreover, the diagnostic techniques tailored to unidimensionality have been refined since Smith's study. In particular, the principal component analysis on the item residuals (Linacre 1998, available, for example, in RUMM 2030, Andrich et al., 2012 or Winsteps, Linacre, 2012) or the g-detect procedure (Kreiner and Christensen, 2004, available in DIGRAM, Kreiner, 2003) offer powerful approaches to investigate dimensionality. Today, there does not seem to be any need for conducting a FA on the raw data prior to a Rasch analysis. In fact, researchers might feel the need to run a confirmatory FA (CFA) *after* the Rasch assessment of a scale in order to use measures in a structural equation model (SEM). However, Rasch measures can be integrated into SEM quite easily. Instructions how to do this can be found in Salzberger (2011).

The sociology of science perspective

From a Rasch perspective, there is no need to run a FA prior to, simultaneously with, or after a Rasch analysis. On the other hand, anyone who has ever tried to publish a Rasch analysis of a scale will have very likely been confronted with the problem of explaining the differences between the RM and FA, felt the pressure to justify the use of the RM, and probably also experienced resistance and refusal. This is where the sociology of science comes in. When one gets into a dispute between paradigms (Andrich 2004), there are at least three different strategies we could pursue, which we might want to call the pure approach, the comparative approach, and the assimilation strategy. First, following the pure approach, the researcher compares the RM and FA at the theoretical macro-level stressing the unique properties of the RM and its relationship to the problem of measurement. The empirical analysis is confined to the RM. Second, the comparative approach aims at exposing empirically the differences between the RM and FA. The RM and FA can be compared at the macro-level, but also at the micro-level. The latter describes, for example, which parameters in the RM correspond most closely to parameters in FA (see Wright, 1996; Ewing et al., 2005). Third, in the assimilation strategy, the Rasch analysis and FA are forced to converge, or at least presented in a way that suggests comparable results based on the RM and FA. Since this strategy downplays the theoretical differences between the RM and FA, a comparison focuses on the micro-level.

The pure approach is probably the most consistent and meaningful path but also the most confrontational. The comparative approach may provide interesting insights but raises the problem of how to argue the superiority of the RM over FA to an audience that does not acknowledge the theoretical underpinnings of the RM. There is a serious threat of falling into the trap of trying to empirically decide whether the RM or FA is better. The assimilation strategy can actually be detrimental to the dissemination of the RM as it easily creates the impression that the RM and FA lead eventually to the same or very similar results. The assimilation strategy can also be pursued unwittingly, particularly when existing scales, originally developed based on FA, are reanalyzed using the RM. Such scales often show a limited variation in terms of item locations. Then the RM as well as FA might exhibit acceptable fit. In addition, the correlation between factor scores, or raw scores, and Rasch measures are typically very high leading to the false impression that the application of the RM generally makes no substantial difference. Issues like invariance, the construct map, the interpretation of measures with reference to items, or targeting, to name just a few, are suppressed.

Conclusions

A Rasch analysis, in principle, hardly benefits from additional input from FA. However, in the interest of acceptance, researchers might feel pressed to incorporate FA into a Rasch paper. Combining Rasch analysis with FA increases the likelihood that non-Rasch researchers (specifically reviewers and editors) become connected with a Rasch paper and that Rasch measurement appears less menacing. At the same time researchers should be cognizant of the potential for misrepresenting the differences between the RM and FA. In any case, it is pivotal to outline the requirements of measurement and to ensure that the Rasch philosophy and the theory of the construct guide the scale development and formation. Then the complementary presentation of results based on FA makes no difference to substantive conclusions. Contributions that aim at a methodological comparison of Rasch measurement and FA are, of course, a different issue.

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A New Human Resource Metric Standard for Investor Guidance

A proposed ANSI-SHRM standard for investor human resource metrics ("ANSI-SHRM 02001.201X Investor Metrics 1stPR DRAFT Standard v1 (040912).pdf") and comments on it are available in the "Metrics and Measures Taskforce (T02)" group at http://hrstandardsworkspace.shrm.org/apps/group_public/document.php?document_id=6504&wg_abbrev=mamt02.

ANSI is the American National Standards Institute, and SHRM is the Society for Human Resource Management. The proposed standard is one of the first of its kind.

One summary proposal concerning the standard was offered by William Fisher, who suggested that the group: “In future revisions of the standard, employ scaling and instrument calibration methods capable of defining invariant units and of then supporting invariant comparisons across different vendors' particular ways of approaching the constructs to be measured.” The full comment was several paragraphs, and published references were provided as resources to be consulted.

Lee Webster, of the standards task force, responded to the proposal on October 5, saying “We appreciate this insight, and agree that more sophisticated measures will (hopefully) be possible in future revisions (as more data become available). Since regular review of standards is a required part of the ANSI process, there will be an opportunity to look at these ideas in depth in the future.”

Standards are widely recognized for their value in simplifying communication and facilitating trade. New standards like that being developed by ANSI and SHRM will be increasingly of interest as intangible assets, such as abilities, health, motivations, and trustworthiness, become ever more central to economic productivity. For background on the role measurement plays in bringing human, social, and natural capital to life, see the references listed below, among others.

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Cronbach's α with the Dimension Coefficient to Jointly Assess a Scale's Quality

Reliability is a necessary, but not sufficient, component of validity (Downing, 2003; Feldt, Brennan, 1989). The dimension coefficient (DC) is, therefore, necessarily incorporated with Cronbach's α to completely and fully describe a scale's characteristics (van der et al., 2003), because not all reliable scales are valid (Cook, Beckman, 2006).

We manipulated data sets containing two types of item length (12 and 20). Each, with 5-point polytomous responses, was uniformly distributed across a ± 2 logit range. This was done for 6 kinds of normally distributed sample sizes (n = 12, 30, 50, 100, 300, and 500) with trait standard deviations (*SD*s) uniformly distributed from 0.5 to 9.5 logits across numbers of misfit items from 0 to 2, all of which misfit items are related to the true score with a zero correlation under Rasch model conditions. A total of 720 (= 2 item lengths × 6 sample sizes × 20 *SD*s × 3 numbers of misfit items) simulation datasets were administered in this study. True-score reliability and dimension coefficients were simultaneously calculated for each simulation data set.

In this case, DCs were temporarily defined by 5 respective approaches, such as Cronbach α, EGA_ratio as Eq.1 that applies the logic of scree plots to propose a ratio by computing the first and second eigenvalues ($R_{12} = \lambda_1/\lambda_2$) with that of the second and third ones ($R_{23} = \lambda_2/\lambda_3$)(Lord, 1980; Divgi, 1980), EGA_angle_ratio as Eq.2 that computes a ratio on angles at the second and third eigenvalues, Rasch loading SD as Eq.3 and Rasch_EGA_ratio as Eq.(4) derived from Rasch PCA on standardized residuals.

$DC = (R_{12}/R_{23})/(1 + (R_{12}/R_{23}))$ Eq. (1)
$DC = (\theta_{12}/\theta_{23})/(1 + (\theta_{12}/\theta_{23}))$ Eq. (2)
DC= 1- Item loading SD Eq. (3)
$DC = (RR_{12}/RR_{23})/(1 + (RR_{12}/RR_{23}))$ Eq. (4)

Type	Sensitivity	Specificity	ROC	95%CI	Cut-off
EGA_ratio	92.46	97.03	0.97	0.94 to 0.98	>0.67
EGA_angle_ratio	94.5	75.20	0.87	0.83 to 0.91	>0.62
Cronbach	62.31	99.01	0.82	0.77 to 0.86	>0.95
Rasch item loading	73.87	76.24	0.82	0.77 to 0.86	>0.54
Rasch_EGA_ratio	74.87	54.46	0.67	0.61 to 0.73	<=0.55

The results were shown in Table 1 using the receiver operating characteristic (ROC) (Fawcett ,2006), in which the area under the curve, sensitivity and specificity for a binary classifier of one and multiple dimensions determined by parallel analysis(Horn, 1965). We found that the EGA_ratio with high sensitivity and specificity can be an approach to compute DC with a cut-off point (>0.67) determining the dimension strength. In our simulation study, the median of DC in Rasch unidimensionality scales without misfit items is 0.94, the highest DC can reach to 0.98.

If an instrument is valid, particularly if the unidimensionality is acceptable, we expect it to be reliable as well. However, an instrument can be both valid and reliable and still not acceptably unidimensional (DC < 0.70). It is also possible to have an instrument with low reliability and low unidimensionality.

This is why we proposed to incorporate Cronbach's α with the DC to jointly assess a scale's quality, and responded to the argument (Sijtsma, 2009) that using Cronbach's α often goes hand-in-hand with the PCA approach in practical test construction, especially when validity is not easily obtained because the true score is unknown.

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A Method of Estimating the Item Parameter from Time on Task

The purpose of this essay is to revisit Chapter 3 of Probabilistic Models (Rasch, 1960) and to consider to what extent the arguments from this chapter might be applied to individual items in a computer based math test. The intention is to look for inspiration from Rasch, but not to follow his methodology exactly.

Hippisley (1999) showed that total completion times from a computer based math test conformed to the (Rasch 1960) reading rate model. However, the assumption of item homogeneity and uniform student speed, rather limits the usefulness of that analysis. In reality, test items are not homogenous, and students do not course through a test at a uniform speed.

Theory

Rasch considered the "speededness" of students reading a text in two ways. For those students who did not complete the text within a prescribed time limit, he treated the number of words read within the time as the stochastic variable. For the students who did complete the test (within the time limit), he treated the time actually taken as the stochastic variable.

For the former group, if λ is mean or expected reading rate, the probability of reading *a* words in time *t* is given as:

$$P\{a \mid t\} = ((\lambda t)^a/a!)e^{-\lambda t} \qquad (1)$$

The probability that no words are read in time *t* is a special case, where N = 0. This reduces down to:

$$P\{0 \mid t\} = e^{-\lambda t} \qquad (2)$$

This expression may be applied to a student tackling a single item in a math test. And while it derives from Expression 1 for reading *rates*, and incorporates λ, (which could be interpreted as an expected item completion rate), the application will be from measuring the *time* during which the item is not completed.

Rasch (1960) broke down λ into two factors. He argued that the ratio of the reading rates between two pupils A and B is interesting if it applies to a number of reading texts. So if in a series of texts, 1, 2, .. i, pupil A reads twice as fast as B:

$$\lambda_{A1} = 2\lambda_{B1}$$

and

$$\lambda_{Ai} = 2\lambda_{Bi}$$

Dividing:

$$\lambda_{Ai}/\lambda_{A1} = \lambda_{Bi}/\lambda_{B1}$$

Generalising:

$$\lambda_{1i}/\lambda_{11} = \lambda_{Ni}/\lambda_{N1} \qquad (3)$$

So the ratio of the mean or expected reading rates of two texts is independent of the pupils. That ratio tells is something about the texts (relative ease of reading or some other applicable descriptor) and it might be given the parameter ε:

$$\lambda_{vi}/\lambda_{v1} = \varepsilon$$

Rearranging:

$$\lambda_{vi} = \lambda_{v1}\varepsilon \qquad (4)$$

Rasch (1960) described the term λ_{v1}, the reading speed of pupil v in a base text, as the person parameter ξv. He also defined the reciprocal of ε as the difficulty δ of a text.

When applying this argument to an individual item in a computer based math test, it should be born in mind that the focus is on a single event. It is impossible to predict exactly when the event will occur, and it is equally impossible to estimate the value of λ from knowing when the event occurred. To overcome this conundrum, a method might be borrowed from natural science.

When physicists consider a sample of radioactive material comprising many atoms, they apply the Law of Large Numbers (Khoshnevisan, 2007), which essentially states that if you run an experiment N times, where N is a very large number, if p is the probability of an event, the number of times the event actually occurs will approximate to Np. From Expression 2 above, the probability of a word not being read, or a math item not being completed, or a radioactive atom not decaying, in time t, is $e^{-\lambda t}$. Studying a sample originally comprising N_0 atoms, is like running an experiment N_0 times. After time t, the approximate number N of atoms, which have not decayed, will be given by:

$$N(t) = N_0 e^{-\lambda t} \qquad (5)$$

The time t_h taken for N to become exactly half N_0 is known as the *half-life* of the material.

$$N_0/2 = N_0 e^{-\lambda th}$$

$$2 = e^{\lambda th}$$

$$\ln 2 = \lambda t_h$$

$$\lambda = \ln 2/ t_h \qquad (6)$$

So if you had a room full of clones all addressing the same item at the same time, you could estimate λ from the time it takes half of them to complete the item. Clones are not easy to come by, but there is another formula from physics which deals with composite radioactive material (L'Annunziata, 2012), and which could be applied to a heterogeneous set of pupils.

In the case of two elements, if the decay rate of Element 1 with N_1 atoms is λ_1 and that of Element 2 with N_2 atoms is λ_2, the combined decay rate λ_c, or number of atoms decaying per unit of time is:

$$-dN/dt = N_1\lambda_1 + N_2\lambda_2$$

In psychometrics, there is usually assumed to be just one pupil of each type, so for two pupils, the combined item completion rate becomes:

$$\lambda_c = \lambda_1 + \lambda_2$$

Reverting to the Rasch notation of Expression 5, if these pupils are addressing item i:

$$\lambda_{ci} = \xi_1 \varepsilon_i + \xi_2 \varepsilon_i$$

$$\lambda_{ci} = \varepsilon_i (\xi_1 + \xi_2)$$

If the same pupils address a second item j:

$$\lambda_{cj} = \varepsilon_j (\xi_1 + \xi_2)$$

Dividing:

$$\lambda_{ci}/\lambda_{cj} = \varepsilon_i (\xi_1 + \xi_2) / \varepsilon_j (\xi_1 + \xi_2)$$

$$\lambda_{ci}/\lambda_{cj} = \varepsilon_i / \varepsilon_j \qquad (7)$$

So the ratio of the combined expected completion rates becomes the ratio of the easiness of the two items, and is independent of the person parameters of the two pupils. A similar argument applies to three or more pupils. Furthermore, the combined expected completion rate can be estimated for each item from the median completion time on each item using Expression 7.

Illustration

Figures 1 and 2 below show for 85 West Australian primary school students, all of whom completed (correctly) the items “4+4”, “3+5”, and “12+8” in a computer based math test, the completion times on item “3+5” against those on item “4+4”, and the completion times on item “12+8” against those on item “4+4”. This triple intersection set arose from a universal set of 14,480 student-item interactions. The settings were informal, with class teachers using the computer based test as a regular class activity, as opposed to a formal exam.

Table 1 shows the median completion times in seconds and an estimate of λ_c for each item. The table also shows ratios of easiness ε and difficulty δ. From the table, if item “4+4” is treated as the base item, item “3+5” seems to be approximately two thirds as easy or 1½ times as difficult, while “12+8” seems to be approximately half as easy or twice as difficult.

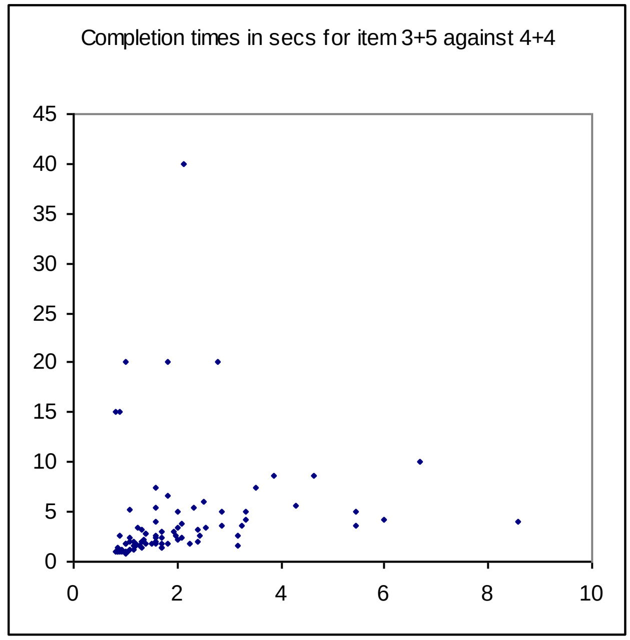

Figure 1.

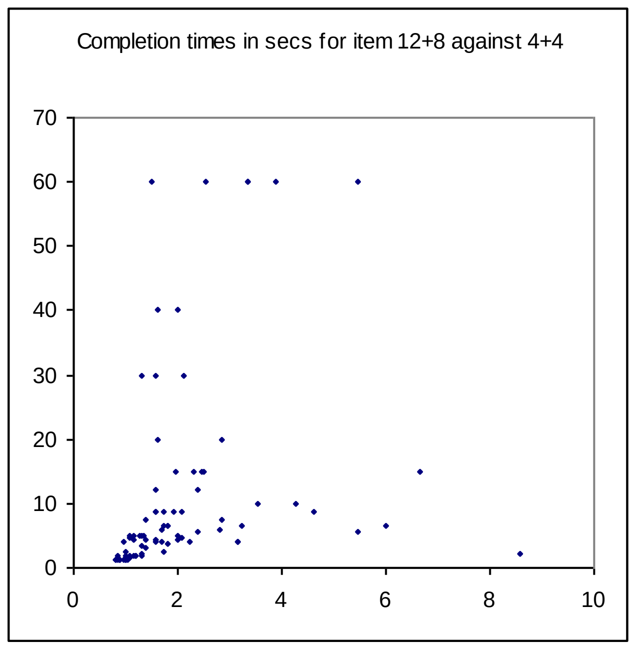

Figure 2.

Item	Median Time (s)	λ	ε ratio	δ ratio
4+4	1.58	0.44		
3+5	2.50	0.28	0.63	1.58
12+8	5.00	0.14	0.50	2.00

The purpose of this essay was to set out a method of estimating the Rasch item parameter from time on task. A method has been laid out, and an illustration has been given. The illustration looked at just three items, all of which had been addressed by the same pupils. Extending the method to cover all of the possible items, which might come up in a simple math test, will either require a very large sample of student-item interactions, or the development of a system, which does not require exactly the same pupils to address every item.

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Is Aberrant Response Behavior a Stable Characteristic of Students in Classroom Math Tests?

Various psychological and demographic characteristics of individuals have been reported to have an association with aberrant response behavior. If indeed they have (or at least some of them do) then one would expect, as suggested by Smith (1986) and Lamprianou (2005), that an individual with an aberrant response pattern may exhibit such behavior in other testing situations too. The research reported here aimed to see if aberrant response behavior is a stable characteristic of high-school students in classroom math tests as expected. That is, whether essentially the same students will misfit in administrations of two different classroom math tests.

In the classroom setting math tests are more relevant, low-stakes, administered by the students' own comfortable-to-be-with teacher and one would perhaps expect less aberrance. This is a completely different context from the high-stakes tests administered in a much stricter and possibly a more stressful environment. At the same time, one would expect some type of aberrance to occur due to carelessness, sleepy behavior, copying, cheating, plodding or guessing.

For the purposes of the study two classroom math tests were used with a sample of 15-16 year old high school students in three different schools in Cyprus. The first test was administered to 635 students and the second to 445 of them. The Rasch Partial Credit Model (PCM) was used for the analyses of the data collected. Misfitting students in both tests were identified with the use of the infit and outfit mean square statistics for six different cut-off values (1.3, 1.4, 1.5, 1.6, 1.8 and 2.0). The hypothesis of no association between misfit in the one test and misfit in the other was investigated with Chi square tests and it was very clearly accepted with p-values much closer to 1.00 than to 0.05. Table 1 shows the observed frequencies and row percentages in brackets for each cut-off value. The last two columns show the chi-square statistics and p-values without the continuity correction and with it in brackets.

Table 1. Chi-square tests for association between misfit in Test 1 and misfit in Test 2

		Test 2			
Cut-off	Test 1	Fitting	Misfitting	Chi-square	p-value
1.3	Fitting	196 (68.1%)	92 (31.9%)	0.514 (0.371)	0.474 (0.542)
	Misfitting	112 (71.3%)	45 (28.7%)		
1.4	Fitting	233 (72.6%)	88 (27.4%)	0.000 (0.000)	0.999 (1.000)
	Misfitting	90 (72.6%)	34 (27.4%)		
1.5	Fitting	261 (76.8%)	79 (23.2%)	0.104 (0.036)	0.747 (0.849)
	Misfitting	79 (75.2%)	26 (24.8%)		
1.6	Fitting	276 (78.0%)	78 (22.0%)	0.000 (0.000)	0.991 (1.000)
	Misfitting	71 (78.0%)	20 (22.0%)		
1.8	Fitting	323 (82.8%)	67 (17.2%)	0.034 (0.000)	0.854 (1.000)
	Misfitting	45 (81.8%)	10 (18.2%)		
2.0	Fitting	345 (86.0%)	56 (14.0%)	0.573 (0.281)	0.449 (0.596)
	Misfitting	36 (81.8%)	8 (18.2%)		

The findings of this study do not support Smith's and Lamprianou's suggestion that aberrance is a stable characteristic of individuals. It is concluded that misfit in the one test is not associated with misfit in the other among high school students taking classroom math tests.

A couple of cautions should be made about this study. First, the test items used were mainly multistep mathematical problems with partial credit awarding for partial success instead of the usual dichotomous items found in the majority of studies on student aberrance. Perhaps it is easier to respond unexpectedly in dichotomous items, especially for high ability students, as reported by Petridou and Williams (2007). Where the answer is marked either right or wrong if a high ability student follows the correct method (as expected) but gives the wrong answer (because of a careless mistake such as a miscalculation, or a miscopy of the right answer) he or she scores 0 and that signals his or her response as unexpected and probably the whole response string as aberrant (especially if the test is short). This is much less likely to happen with multistep problems. If such a mistake occurs, on the last stages of the solution process, the student will get most of the marks on that item and the answer will not be considered unexpected. Second the low stakes status of the tests linked to the administration procedure, with the familiar classroom setting may make the test takers feel more relaxed and perform more as expected than in a stricter and less familiar environment.

The finding of this study, explored further in Panayides' (2009), lead to the following intuitive conclusion: In classroom math tests, although misfits do occur, they do not predict misfits in other tests and are not dependent on psychological or demographic characteristics of the test-takers.

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Harvey Goldstein's Objections to Rasch Measurement: A Response from Linacre and Fisher

Let us start by considering Harvey Goldstein (HG, 2012, p.153):

HG: "The specific literature on the 'Rasch' model, a particularly simple item-response model, is ... insistent that only a single dimension is needed in any given application,"

JML comment: The number of dimensions needed, or encountered, in a given application depend on the application, but, whenever we talk about "more" or "less" of something, we have declared the "something" to have the properties of a dimension. The goal of the Rasch model is to quantify that dimension in terms of additive units of "more"-ness. The complexity of the Rasch model matches this task.

WPF comment: Quantification is inherently linear along a single dimension of more and less. If quantification is desired, isolating those aspects of a construct that exhibit consistent variation is essential.

HG: "The specific literature on the 'Rasch' model .... displays a general unwillingness to explore further (see Goldstein 1980 for an illustrative example)."

JML comment: Rasch analyses are unusual in that every person, demographic group, item, item response option, even each individual response, can be reported with fit statistics, estimates and other indicators, as appropriate. Routine exploration of any dataset includes searching for secondary dimensions in the data, and determining their impact on the empirical functioning of the intended dimension. The depth and complexity of Rasch analysis has advanced considerably since 1980. For instance, the User Manual for BICAL, the leading Rasch software in 1980, was 95 pages of text. BICAL has about 2,000 lines of computer code. An equivalent Rasch program in 2012, Winsteps, has a User Manual with 615 pages of text and has more than 70,000 lines of computer code.

WPF comment: The specific literature that refers to Rasch's work is wide ranging in the explorations of the infinite ways in which constructs can interact, overlap, or display anomalous features. Karabatsos (2003), for instance, examines 36 different ways of evaluating inconsistencies in person measures. In addition, a wide range of Rasch models for item bundles or testlets, multidimensional collections of constructs, multilevel models of group-level effects, and multifaceted situations of many kinds have emerged in the last 30 years.

HG: "proponents of this model regard the model as paramount and suggest that data should be constructed or modified to satisfy the model's assumptions."

JML comment: Social Scientists, indeed scientists of all types, construct or modify data to meet their intentions. For instance, Census Bureaus construct the data they want by writing appropriate questions. Analysis of Census data often requires that the data be modified, because the analytical question does not exactly match the question on the Census form.

Currently "data mining" methodology is in vogue and considered to be highly successful. Here are its stages (Fayyad et al., 1996): (1) Data Selection, (2) Data Pre-processing, (3) Data Transformation, (4) Data Mining, (5) Interpretation/Evaluation. Rasch methodology uses the same stages, but with (4) Rasch analysis. Stages (1) and (2) correspond to data construction and modification. A difference is that Rasch analysts tend to be more methodical and overt about their data procedures.

WPF comment: HG's objection is written in a grammatically correct English sentence. This sentence and manner of communication prioritizes a model of a competent English reader able to understand written text. HG, like most other proponents of this model, regard it as paramount and assume that readers will be able to construct or modify data to satisfy the model's assumptions. A measurement model is no different. Instruments are texts that are written, read and interpreted using the same cognitive operations employed in any act of reading. HG would no more attempt written communication in terms of a model that allows ungrammatical constructions, mixed languages and orthographies, or stray marks than measurement should be attempted in terms of models that legitimate just any kind of data. GIGO.

HG: "Thus, Andrich (2004) claims that this model satisfies the conditions of 'fundamental measurement' and as such attains the status of measurement in the physical sciences"

JML comment: From a practical perspective, most measurement in the physical sciences is based on additivity, "one more unit is the same amount extra, no matter how much there already is." Additivity can be demonstrated for Rasch parameter values (Rasch measures) (Wright 1988), so Rasch measures have the practical status of physical measures.

WPF comment: Measurement in physics is often misconstrued as primarily a matter of accessing concrete objects. On the contrary, the laws of science project unrealistic and unobservable phenomena, like balls rolling on frictionless planes, or objects left entirely to themselves with no external influence, or a point-like mass swinging on a weightless string. Rasch models are exactly like models in physics in this respect of positing unobservable ideals that serve as heuristic guides to inference and decision making.

HG: "– a view about measurement in the social sciences that in a slightly different context Gould (1981) has labelled 'physics envy'."

JML comment: "Overcoming Physics Envy" (Clarke & Primo, 2012) begins "How scientific are the social sciences? Economists, political scientists and sociologists have long suffered from an academic inferiority complex: physics envy. They often feel that their disciplines should be on a par with the 'real' sciences and self-consciously model their work on them, using language ('theory,' 'experiment,' 'law') evocative of physics and chemistry."

Yes, Rasch analysts also share this feeling. But is it a bad feeling? Haven't "theory," "experiment," "law" generated 400 years of obvious progress in physics and chemistry? Would social science be possible without theories and hypotheses to guide our thoughts, experiments to verify our conclusions, laws (observed regularities) to encapsulate those conclusions into communicable and useful forms? It is the same with measurement. "How much?" is a basic question in both "real" science and social science. Additive measures of well-defined variables are the most straight-forward way for us to think about, communicate and utilize "much"-ness.

"Overcoming Physics Envy" ends "Rather than attempt to imitate the hard sciences, social scientists would be better off doing what they do best: thinking deeply about what prompts human beings to behave the way they do."

But "thinking deeply" is exactly what Rasch facilitates. The bulk of the raw data is segmented into well-behaved, understandable dimensions on which carefully-thought-out defensible inferences about human beings can be based. The ill-behaved remnants of the raw data are perplexing, perhaps inexplicable. We can think deeply about these remnants and perhaps generate new insights from them about human behavior, but these confusing remnants do not impede us from making progress.

WPF comment: Many social scientists have long been doing what they do best. Beginning from the emergence of qualitative methods in the 1960s and 1970s, there has been less and less concern with imitating any other field, while more and more effort has been invested in creative thinking. Recent studies of model-based reasoning in science (for instance, Nersessian, 2006, 2008) show that scientific thinking is not qualitatively different from any other kind of thinking. The goal is not to imitate physics or any one field, but to think productively in a manner common to all fields. Rasch (1960) explicitly draws from Maxwell's method of analogy, which is exactly the example Nersessian (2002) uses to illustrate model-based reasoning (Fisher, 2010).

Now let us consider Goldstein (2010), his response to Panayides et al. (2010). Goldstein asserts that the Rasch "model is inadequate, and that claims for its efficacy are exaggerated and technically weak." Here is the evidence he presents in support of this generalization.

HG: Around 1980, in the United Kingdom, "the advocates of using Rasch, notably Bruce Choppin, had a weak case and essentially lost the argument. It was this failure to make a convincing case that led to the dropping of the use of this model for the [United Kingdom]."

Rasch-related Coming Events

Dec. 10-12, 2012, Mon.-Wed. In-person workshop: Intermediate Rasch (A. Tennant, RUMM), Leeds, UK, www.leeds.ac.uk/medicine/rehabmed/psychometric

Jan. 4-Feb. 1, 2013, Fri.-Fri. Online workshop: Practical Rasch Measurement – Core Topics (E. Smith, Winsteps), www.statistics.com,

March 25-27, 2013, Wed.-Fri.. In-person workshop: Introductory Rasch (A. Tennant, RUMM), Leeds, UK,

Apr. 27 – May 1, 2013, Sat.-Wed. AERA Annual Meeting, San Francisco, CA, www.aera.net,

May 15-17, 2013, Wed.-Fri. In-person workshop: Introductory Rasch (A. Tennant, RUMM), Leeds, UK,

May 20-22, 2013, Mon.-Wed. In-person workshop: Intermediate Rasch (A. Tennant, RUMM), Leeds, UK,

May 31-June 28, 2013, Fri.-Fri. Online workshop: Practical Rasch Measurement – Core Topics (E. Smith, Winsteps), www.statistics.com,

July 5-Aug. 2, 2013, Fri.-Fri. Online workshop: Practical Rasch Measurement – Core Topics (E. Smith, Winsteps), www.statistics.com.

JML comment: Around 1980, a convincing case could not be made for any psychometric methodology, as my employer at the time, Mediax Associates, discovered. However, indications were more hopeful for Rasch than for any of its competitors. Linacre (1995) demonstrates that the deficiencies in the British educational system, confirmed by Bruce Choppin's application of Rasch methodology, were crucial in its rejection.

HG: "the essence of the criticisms remains and centres around the claim that the model provides a means of providing comparability over time and contexts when different test items are used."

JML comment: In 1980, the empirical evidence for comparability was weak, even though the theoretical basis was strong. By 1997, the empirical evidence was also strong (Masters, 1997). By 2012, so many testing agencies have maintained comparability for many years by using Rasch methodology that it is now routine.

WPF comment: Bond (2008) reports one such routinely maintained basis for comparability. Re-analysis of data from items used on tests over periods of 7 to 22 years at one major testing agency showed that "correlations between the original and new item difficulties were extremely high (.967 in mathematics, .976 in reading)." Bond continues, saying "the largest observed change in student scores moving from the original calibrations to the new calibrations was at the level of the minimal possible difference detectable by the tests, with over 99% of expected changes being less than the minimal detectable difference."

HG: "Misconceptions and inaccuracies. First, ... all claims about item characteristics being group-independent and abilities being test-independent, can be applied to [Classical, IRT and Rasch] types of model."

JML comment: Here is an experiment. Simulate a dataset of 1000 persons and 200 items according to each of the models. Split each dataset in two, the 500 higher-scoring persons, and the 500 lower-scoring persons. Analyze each pair of resulting datasets separately. To investigate group-independence, cross-plot the pairs of item difficulty estimates. Do they follow a statistically straight line? No, except for Rasch models or models that approximate Rasch models.

Now split the original datasets in two again, the 100 higher-scored items, and the 100 lower-scored items. Analyze the pairs of resulting datasets separately. To investigate test independence, cross-plot the two sets of person ability estimates. Do they follow a statistically straight line? No, except for Rasch models and estimation procedures that impose the same person distribution on both datasets. In summary, all claims cannot be applied to all models. Only Rasch models support the claims.

HG: "Secondly, ... a 2-dimensional set of items (representing different aspects of mathematics) could actually appear to conform to a (unidimensional) Rasch model, so that fitting the latter would be misleading."

JML comment: Yes, a dataset that balances two distinct dimensions can appear unidimensional on first inspection, so current Rasch best-practice is to include an investigation of the dimensionality of a dataset. All empirical datasets are multidimensional to some extent. In this example, the decision must be made as to whether the different aspects of mathematics (say, arithmetic and algebra) are different enough to be considered different "dimensions" (say, for the purpose of identifying learning difficulties) or are merely different strands of a superordinate dimension (say, for the purpose of Grade advancement).

WPF comment: Yes, Smith (1996) illustrates the value of a Principal Components Analysis of Rasch model residuals, showing its value in detecting multidimensionality when two or more constructs are roughly equally represented in an item set. PCA's strength in this situation is complemented by the sensitivity of the usual fit statistics when items primarily represent a single construct and only a few are off-construct or otherwise problematic.

HG: "Thirdly, the authors claim that there are no sample distributional assumptions associated with the Rasch model. This cannot be true, however, since all the procedures used to estimate the model parameters.... necessarily make distributional assumptions."

JML comment: Yes, different estimation methods make different assumptions. For instance, many Rasch maximum-likelihood estimation methods (including CMLE, JMLE, PMLE) make no assumptions about the distributions of the person abilities and item difficulties, but do assume that the randomness in the data is normally distributed. This assumption is routinely validated using fit statistics.

WPF comment: The term "assumption" here is misused. An assumption is something taken for granted, something left unexamined on the basis of its status as something in no need of attention. What HG refers to as assumptions are in fact the very opposite. What distinguishes the art and science of measurement from everyday assumptions about what are matters of fact is that very close attention is paid to the requirements that must be satisfied for inference to proceed.

HG: "Fourthly, ... the authors.. claim that a 'fundamental requirement' for measurement is that for every possible individual the 'difficulty' order of all items is the same. This is ... extremely restrictive. ... I also find it difficult to see any theoretical justification for such invariance to be a desirable property of a measuring instrument."

JML comment: The difficulty hierarchy of the items defines the latent variable. The easy items define what it means to be low on the latent variable. The hard items define what it means to be high on the latent variable. We measure a person's ability on a latent variable (for instance, "arithmetic") in order to make inferences about that person's arithmetic performance. If the definition of the latent variable changes depending on the person's ability level, then we cannot make general statements such as "division" is more difficult than "addition" (Wright, 1992). We must add the impractical restrictive phrase, "for people at such-and-such ability level". The inferential value of the latent variable is severely diminished.

WPF comment: Being unable to see any theoretical justification for invariance as a desirable property of a measuring instrument belies fundamental misconceptions of what instruments are and how they work. Invariance is the defining property of instruments, no matter if they are musical, surgical, or measuring. Without invariant measures, orchestras and laboratories would be impossible. "The scientist is usually looking for invariance whether he knows it or not. ... The quest for invariant relations is essentially the aspiration toward generality, and in psychology, as in physics, the principles that have wide applications are those we prize (Stevens 1951, p. 20). Perhaps HG terms invariance restrictive because he misconceives it in some kind of absolute way, as Guttman did. In actual practice, the uncertainty ranges within which items fall vary across different kinds of applications. Screening tolerates more uncertainty than accountability, which tolerates more than diagnosis, and which can in turn tolerate more than research investigations of very small effect sizes.

HG: "Fifthly, the authors do not seem to appreciate the problem of item dependency. .... There are all kinds of subtle ways in which later responses can be influenced by earlier ones."

JML comment: An advantage of Rasch methodology is that detailed analysis of Rasch residuals provides a means whereby subtle inter-item dependencies can be investigated. If inter-item dependencies are so strong that they are noticeably biasing the measures, then Rasch methodology supports various remedies. For instance, it may be advantageous to combine the dependent items into polytomous super-items (so effectively forming the items into testlets).

WPF comment: One of the significant reasons for requiring unidimensionality and invariance is, in fact, to reveal anomalous local dependency. "To the extent that measurement and quantitative technique play an especially significant role in scientific discovery, they do so precisely because, by displaying significant anomaly, they tell scientists when and where to look for a new qualitative phenomenon" (Kuhn, 1977, p. 205). As another writer put it, expect the unexpected or you won't find it (van Oech, 2001). If you begin with the intention of modeling dependencies, every data set and every instrument will be different, and all of the differences distinguishing them will be hidden in the modeled interactions. The predominance of modeling of this kind is precisely why the social sciences have made so little progress. Real progress will be made only when we implement uniform measurement standards capable of supporting the kind of distributed cognition common in language communities (Fisher, 2012), whether one defines those communities in terms of English or Mandarin, or in terms of Newton's Second Law and the *Systeme Internationale des Unites*.

HG: "Sixthly, ... This comes dangerously close to saying that the data have to fit the preconceived model rather than finding a model that fits the data. It is quite opposed to the usual statistical procedure whereby models (of increasing complexity) are developed to describe data structures. Indeed, the authors are quite clear that the idea of 'blaming the data rather than the model' is an important shift from standard statistical approaches. In my view that is precisely the weakness of the authors' approach."

jMetrik

jMetrik is a free and open source computer program for psychometric analysis. jMetrik is available for download from www.ItemAnalysis.com. It features a user-friendly interface, integrated database, and a variety of statistical procedures. The interface is intuitive and easy to learn. It also scales to the experience of the user. New users can quickly learn to implement psychometric procedures though point-and-click menus. Experienced users can take advantage of the jMetrik command structure and write command files for executing an analysis.

jMetrik's embedded database increases productivity by providing a common data format for all of its methods. There is no need to reformat or reshape data for each procedure. The database is the primary mechanism for data management. There is virtually no limit to the sample size or number of tables that can be stored in the database. Users are only limited by the amount of storage on their computer. After importing data into jMetrik, users can create subsets of data by selecting examinees or variables. Users can also create new tables by saving the results of an analysis in the database for further processing. Statistical methods available in jMetrik include frequencies, correlations, descriptive statistics and a variety of graphs.

Psychometric methods include classical item analysis, reliability estimation, test scaling, differential item functioning, nonparametric item response theory, Rasch measurement models, and item response theory linking and equating. New methods are added to each new version of the program.

jMetrik is a pure Java application. It runs on Windows, Max OSX, and Linux operating systems. Installation files include the needed version of Java Virtual Machine. An additional system requirement is 256MB of available memory.

JML comment: What is here perceived to be "dangerous" and "weakness", most of Science perceives to be necessary and strength. In general throughout Science, a theory is constructed that usefully explains and predicts important aspects of the data. This theory then becomes the screen through which future data are validated. Only if some future data cannot be coerced to conform to this theory, and those data are shown to be valid, is this theory bypassed in favor of some other theory and perhaps only for those data. Rasch theory is useful in that it constructs additive unidimensional measures from ordinal data. CTT and non-Rasch IRT may provide better statistical descriptions of specific datasets, but the non-linearity of their estimates and their sample-distribution-dependent properties render them less useful for inference.

WPF comment: Again, by writing in English and on a technical subject, HG must require readers who fit his preconceived model of the particular kind of person able to understand his text. When he takes the measure of the situation and puts it in words, he makes no effort whatsoever to find a model for his text that will fit any person at all who happens to approach it. He very restrictively requires readers capable of reading English and of comprehending somewhat technical terms. He gladly sets aside the vast majority of the world population who are unable to comprehend, or who are merely uninterested in, his text. In positing the Pythagorean theorem or Newton's laws, we do exactly the same kind of thing, focusing our attention on the salient aspects of a situation and ignoring the 99.999% of the phenomena that do not correspond. Our failure to do this more routinely in the social sciences says more about the way we misunderstand language, cognition, and our own instruments than it does about any kind of supposed shortcoming in Rasch theory.

HG: "Finally, ... The old Rasch formulation is just one, oversimple, special case. All of these models are in fact special kinds of factor analysis, or structural equation, models which have binary or ordered responses rather than continuous ones. As such they can be elaborated to describe complex data structures, including the study of individual covariates that may be related to the responses, multiple factors or dimensions, and can be embedded within multilevel structures."

JML comment: Rasch models construct additive measures (with known precision) from binary or ordered responses. Additive measures are ideal for further statistical analysis. Far from being obsolete, Rasch models are seen to be useful building-blocks on which to build elaborate statistical structures.

WPF comment: HG's observation assumes that measurement is primarily achieved by means of data analysis. But once an instrument is calibrated, and the item estimates persist in their invariant pattern across samples and over time, does not further data analysis become exceedingly redundant? Only the most counter-productive and obstructionist kind of person would resist the prospect of capitalizing on the opportunity to make great efficiency gains by fixing the unit at a standard value. Yes, Rasch mixture, multilevel, multifaceted, item bundle, etc. models are highly useful, but an important goal is to create a new metrological culture in the social sciences. Qualitative and quantitative data and methods need to be blended in the context of instruments tuned to the same scales. Only then will we find paths to new ways of harmonizing relationships.

HG: "Attempting to resurrect the Rasch model contributes nothing new."

JML comment: Only in the UK has the Rasch model needed resurrection. However, "attempting to resurrect the Rasch model" forces us to reconsider the philosophy underlying Social Science. Is Social Science to become exclusively qualitative with an endless accumulation of suggestive case studies but no counts of anything? Is Social Science to become exclusively quantitative with its focus solely on summary statistics and arcane descriptive models? Or is Social Science to become a synergistic blend of qualitative and quantitative? This is the ideal toward which Rasch methodology strives as it attempts to construct meaningful, sometimes new, qualitatively-defined unidimensional variables out of counts of inevitably messy ordered observations.

WPF comment: The point is to be able to persist in questioning, to continue the conversation. Statistical models can sometimes describe data to death, meaning that they become so over-parameterized that nothing of value can be generalized from that particular situation to any other. All models are wrong, as Rasch (1960, pp. 37-38; 1973/2010) stressed. But even though there are no Pythagorean triangles in the real world, they still prove immensely useful as heuristic guides to inference in tasks as concrete as real estate development, titling, and defending property rights. If we can resist the pressures exerted by HG and others bent on prematurely closing off questioning about potential general invariances, we may eventually succeed in creating real value in social science. But if we instead focus *only* on ephemeral local specifics inapplicable beyond their immediate contexts, we will continue to be subject to aspects of our existence that we do not understand.

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The Web Index: Gender Bias Findings from the Rating Scale model

The Web Index (WI), developed by the World Wide Web Foundation and launched in September 2012, aims at measuring the impact of the Web on people and nations. It is computed for 61 countries worldwide and consists of 85 indicators across 7 components: Communications Infrastructure, Institutional Infrastructure, Web Content, Web Use, Political Impact, Economic Impact and Social Impact of the Web (Farhan et al., 2012). The WI combines both existing hard data, from official providers, and new data gathered via a multi-country questionnaire - primary data - specifically designed by the Web Foundation and its advisers. The questionnaire was submitted to country experts for the first release of the Index. It consists of 63 questions each on a 1 to 10 scale, positively oriented with respect to the Web impact level. The scores given by country experts were checked and verified by a number of peer and regional reviewers for each country.

Table 1: List of original questions

Question	ID
To what extent are **boys** trained in the use of computers?	Q9a
To what extent are **girls** trained in the use of computers?	Q9b
To what extent are **girls** encouraged to focus on science and technology?	Q9c
To what extent are **boys** encouraged to focus on science and technology?	Q9d
To what extent does the government publicize the importance of access to the Web to all the population?	Q9e
To what extent does the government publicize the importance of access to the Web specifically for **women**?	Q9f
To what extent are there gov't programs specifically focusing on funding training for their staff in ICT use?	Q9g
To what extent are there gov't programs specifically focusing on funding training for their **female** staff in ICT use?	Q9h
In your country, to what extent are there **female** "role models" in the ICT field?	Q9i
In your country, in tertiary education, what proportion of ICT graduates are **women**?	Q9l
To what extent does the gov't impose restrictions on access to websites (censorship)?	Q10
To what extent are there laws against cyber crime in your country?	Q13
To what extent would you consider your country to be ranking amongst the World's best in training computer engineers?	Q16
Does the gov't have a specific Open Gov't Data initiative?	Q25

Rating Scale Model for primary data

Primary data are the backbone of the WI. Our analysis on primary data aimed at highlighting possible improvements of the questionnaire used to collect primary data and detecting specific behaviours both for countries and questions. To this purpose the Rating Scale model is employed for each of the components separately (Annoni et al., 2012). The focus here is on one WI component, the Institutional Infrastructure, which "looks at extent to which institutions, organizations and government support and promote the Web access, and the extent to which information about their organizations is made available on the Web" (Farhan et al., 2012, pg. 23). This component is then of fundamental importance in measuring the Web

Impact and is the only one including eight questions aimed at highlighting possible gender imbalances. Table 1 lists of questions included in this component.

The Rating Scale Model revealed that questions Q10 and Q25 had poor fit characteristics, with infit and outfit statistics > 2. These questions were excluded from further analysis. In the final Rating Scale Model all the questions showed good fit statistics. The Rasch model explains about 62% of data variability and evidences a clearly unidimensional construct. The response structure organised in a ten category scale was evidenced to be appropriate, as no country shows a notable unexpected pattern of answers, confirming that the questionnaire has been always scored by experts at their best. However, DIF analyses reveal a gender bias issue.

Is there a gender-bias issue?

By comparing the locations of questions Q9a - Q9b, which describe computer training for boys and girls, and Q9c - Q9d, describing science and technology training for girls and boys, we notice that the ones referring to boys (Q9a and Q9d) are easier than their female counterparts. Moreover, the most difficult questions all refer to the female situation, clearly raising a gender bias issue.

Table 2: Countries included in the analysis classified according to GDP per capita in PPS.

GDP per cap ≤ P_{25}	P_{25} < GDP per cap ≤ P_{50}	P_{50} < GDP per cap ≤ P_{75}	GDP per cap ≥ P_{75}
Bangladesh	Brazil	Argentina	Australia
Benin	China	Chile	Canada
Burkina Faso	Colombia	Israel	Finland
Cameroon	Ecuador	Italy	France
Ethiopia	Egypt	Kazakhstan	Germany
Kenya	Ghana	Korea (Rep. of)	Iceland
Mali	India	Mauritius	Ireland
Nepal	Indonesia	Mexico	Japan
Nigeria	Jordan	New Zealand	Norway
Pakistan	Morocco	Poland	Qatar
Senegal	Namibia	Portugal	Singapore
Tanzania	Philippines	Russia	Sweden
Uganda	South Africa	Spain	Switzerland
Yemen	Thailand	Turkey	United Kingdom
Zimbabwe	Tunisia	Venezuela	United States
	Vietnam		

A DIF analysis is also performed on groups of countries defined on the basis of GDP per capita in PPS (according to the International Monetary Fund). Countries are classified according to the quartiles of the distribution of the country average GDP per capita over the period 2008-2012 (Table 2). The analysis shows that question Q9d (boys' training on science and technology) performs differentially for the richest (> P_{75}) and the poorest countries (≤ P_{25}). Question Q9l (share of female ICT graduates) performs differentially for the richest countries (Table 3). Scores on question Q9d, higher than expected in the poorest countries, indicate that in these countries boys are particularly encouraged to focus on science and technology. The opposite occurs for the richest countries, suggesting a gender-bias issue which is more relevant in the poorest countries than in the richest ones. Scores on question Q9l are lower than expected in richest countries, indicating a lower attitude of girls towards ICT degrees in these countries than expected by the model. This suggests that even in the richest countries the ICT sector needs to attract more women, with all the implications that this may have. A participation discrepancy between genders in science, technology and ICT topics has been recently highlighted in the European Member States (DG- EMPL, 2010), where stereotypical assumptions about IT-related jobs still play a significant role. Does our analysis agree with this? Awkwardly, we think so.

Table 3: DIF outcomes for different groups of countries.

Component	Country Group	Q	Q-Difficulty	Scores
Institutional infrastructure	Poorest	Q9d	< than expected	> than expected
	Richest	Q9d	<than expected	> than expected
	Richest	Q9l	<than expected	> than expected

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